Limits of Arbitrage:
An Econometric Analysis of Evidence Based on Hedge Fund Returns

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Research Question:

Do hedge fund invest more or less aggressively during periods when underlying markets are more volatile?

Contribution:

To develop and apply an econometric methodology to employ hedge fund returns to approximate relative changes in hedge fund investment, in order to document whether hedge funds *reduce* or *increase* their exposure during periods when underlying return drivers become more volatile.

Why should this matter? …

Theory (and Some Anecdotes), But No Systematic Empirical Tests of the Evidence ...Why Not?

SV1997 suggested testing whether hedge fund increase or reduce investment during periods of high market volatility:

Empirical Question: Do hedge funds *reduce* or *increase* their notional investment exposure when shocks occur?

HOWEVER, certain econometric challenges exist to empirically documenting changes in hedge fund investment:

(a) The most reliable available data is limited to monthly hedge fund returns.

(b) Active trading by hedge funds results in higher residual errors and lower statistical significance relative to buy-and-hold investment funds (e.g. mutual funds).

(c) High cross-sectional correlation between contemporaneous returns across a sample of hedge funds often results in multicollinearity.

A Clue???

The Notion of "*Risk-Based Leverage*" ("*RBL*")

RBL is the ratio of the riskiness of the investment exposure to the equity capital of a fund.

RBL is accepted in practice as an accurate approximation of the effective, aggregate investment exposure of a fund.

RBL reflects the relationship between the risk of the fund's total investments and the fund's capacity to absorb that risk.

"*Sound Practices for Hedge Fund Managers*" (February 2000) – A policy study prepared for regulators by five of the largest hedge funds:

> "Hedge Fund Managers must recognize that leverage is important…i.e., leverage influences [the magnitude and rate of] changes in the value of the portfolio due to changes in market risk, credit risk, or liquidity risk factors. Consequently, the most relevant measures of leverage are "risk-based" measures… Hedge Fund Managers should assess the degree to which a Hedge Fund is able to modify its risk-based leverage in periods of stress or increased market risk."

- In the absence of more direct tests involving the actual ratio of the notional investment to the capital of hedge funds, *RBL* can be approximated by using regression to estimate the covariation between hedge fund returns and factors underlying those returns.

- Furthermore, the magnitude of the change in a fund sample's *RBL* relative to the sensitivity of its returns to increased volatility should be measurable.

RBL Defined

Leverage serves as the multiplier between the risk and return of an investment and the corresponding risk and return of the equity capital underlying the investment As such, Breuer (2000) defines leverage as the elasticity of the value of the equity capital with respect to the value of the [investment], such that the return on hedge fund equity capital as it corresponds to the return on investment of the hedge fund

$$return_{fund} = return_{Investment} * Leverage$$

This implies that a one percent change in the return on the investment translates into L percent points of return on equity:

$$\sigma(return_{fund}) = \sigma(return_{Investment}) * Leverage \text{, where } Leverage = Assets/Equity$$

If what Breuer calls "Assets" is defined as follows,

$$Assets = (m * I)/E \text{ ,}$$

where *m* represents the units of investment *I*, expressed in notional amounts equal to or greater than $1, and *E* is the endowment of equity capital equal to $1, then the return of the fund (and the risk of the fund's return) increases or decreases, with either:

A. an increase or decrease in the investment return (and the risk of the investment return), or

B. with the notional amount of investment supported by the equity capital of the fund.

Hence, the change in *RBL* measures the change in the risk of the equity capital of the fund resulting either from changes in the riskiness of the investment or from changes in the actual leverage assumed by the fund.

Hypotheses

Based upon observed changes, the cross-sectional variation between hedge fund returns and the standard deviation of underlying common factors in returns:

Hypothesis #1:
During those monthly periods when common factors underlying hedge fund returns exhibit increased volatility (i.e. shocks), do hedge funds exhibit losses (or gains)?

Hypothesis #2:
During those monthly periods when "common factors" underlying hedge fund returns exhibit shocks, do hedge funds in general decrease (or increase) *RBL*, i.e. their exposure to those factors?

Hypothesis #3:

A. Are hedge funds with higher SD of preshock returns more or less constrained before, during, and after shocks than hedge funds?
B. Do less constrained funds exhibit higher RBL prior to and during shocks?
C. Do the returns of hedge funds with a higher standard deviation (SD) of preshock returns exhibit *negative* or *positive* covariance with the increased volatility of underlying spreads or common factors?
D. What about the returns of hedge funds with a lower SD of preshock returns?

Hypothesis #4:

A. During those monthly periods when common factors underlying hedge fund returns exhibit shocks, do the *less constrained* hedge funds (those with lower SD of returns) reduce *RBL* more or less than the *more constrained* hedge funds?
B. Which constrained hedge funds exhibit higher returns (or lower losses) during shocks, the more or less constrained funds?

Two Sets of Tests

The first test to be performed with a (more) homogenous sample of hedge fund returns
…Why fixed income arbitrage strategies?

Motivation:

- 98% of the variation in bond value explained by factors related to the term structure of interest rates -- Litterman and Scheinkman (1991)

- The liquidity premium implicit in the yield of a bond well-established as a factor underlying the relative-value of comparable fixed income instruments (Buraschi and Menini, 2001; Krishnamurthy, 2001)

- Commonly noted and generally accepted that fixed income hedge funds will often engage in arbitrage strategies based on the liquidity premium implicit in bond yields (Berens and Friend, 1997; Dunbar, 1999; Lowenstein, 2000)

- Volatility changes are also well-documented as determinants in fixed income returns (Litterman, Scheinkman and Weiss, 1991)

- Both academic and industry research suggest the existence of relative-value trading opportunities that hedge funds attempt to exploit (Longstaff, Santa Clara and Schwartz, 2000a,b; Kocic, 2000, 2001; Berens and Friend, 1997)

For a broader, more diverse sample of hedge fund returns, does the same result hold?

Empirical Design

Description of Data Samples: What criteria was used to select and sort funds for each of the two samples?

For each of the two samples, what was the basis for sorting monthly hedge fund returns in Preshock, Shock, and Postshock periods?

Empirical Design (Cont'd)

The general empirical framework for both sets of tests,…

(1) $$return_{n,t} = \alpha + \Sigma_i \left[b_i X_{i,t} \right] + u_t$$

where,

$\alpha = 0$ (Why constrain for the intercept to be $\alpha = 0$?)

n = the nth fund in the sample of 24 hedge funds

b = the factor coefficient where $\Sigma|b|$ is a proxy for an estimate of risk-based leverage

X = a proxy for factors based on monthly changes in specified fixed income spreads

i = where $i = 1,2,3$ each represent a spread proxy related to market, liquidity or credit risk.

t = month in the sample from February 1997 to March 2000

is an extension of the multi-factor regression framework introduced by Sharpe (1992) and applied by FH1997, based on the APT (Ross, 1976)

Empirical Design – Part I (Cont'd)

The first set of tests apply the multifactor regression framework to hedge funds engaged in fixed income investment strategies…

One-factor regression for returns of hedge funds engaged in fixed income investing:

(2) $return_{n,t} = \alpha + b_{1,t}[SWAPS30_2_t] + \xi_t$, where $\alpha = 0$

Three-factor regression for returns of hedge funds engaged in fixed income investing:

(3) $return_{n,t} = \alpha + b_{1,t}[SWAPS30_2_t] + b_{2,t}[ON/OFF_30_t] + b_{3,t}[STRADDLE3M_10Y_t] + \eta_t$, where $\alpha = 0$

Empirical Design – Part I (Cont'd)

Fixed Income Arbitrage Hedge Fund Sample:

- Monthly returns reported in TASS for twenty-four (24) hedge funds
- Report returns for every month from February 1997 through March 2000
  (All Survivors of LTCM Crisis – Why???)
- Monthly returns sorted into preshock, shock, and postshock months based on the standard deviation of underlying spreads for the month in question

Figure I. Fixed Income sample fund returns compared to TASS index fund return and SD of Swap Term Spread

Sample Period Jan 97- Mar 00	Mean One-Month Return Fixed Income Arb Sample	Mean One-Month Return Fixed Income Arb Index	Mean One-Mth Std Dev 30Yr vs. 2 Yr Swaps
Preshock	0.9501	0.9125	0.0541
Shock	-0.7924	-0.3914	0.0759
Postshock	0.5628	0.5969	0.0440

Source: TASS Hedge Fund Index, Reuters

Empirical Design – Part I (Cont'd)

Variable Selection for a Regression Model of Hedge Funds Engaged in Fixed Income Investing:

Figure II. Three Factors Selected as Proxies for Generic Fixed Income Hedge Fund Trading Exposures

RHS Variables* 'Factors'	**1-Mth Std Deviation *'SWAPS30_2'***	**1-Mth Std Deviation *'ON/OFF_30'***	**1-Mth Daily Return *'STRADDLE3M_10Y'***
Preshock	0.0541	0.0098	0.0146
Shock	0.0759	0.0181	0.0545
Postshock	0.0440	0.0164	0.2826

Source: TASS Hedge Fund Index

*Mean SD reported for *'SWAPS30_2'* and *'ON/OFF_30'*; Mean one month (cumulative daily rolling) returns for 30-Day European options for *'STRADDLE3M_10Y'*

Exhibit A. Common Determinants Underlying Risk-Based Leverage for Fixed Income Hedge Funds

Factor in Return	**Underlying Spread**	**Risk Factor**	**Spread Risk Proxy**
Term premium	$Swaps_{10yr} - Swaps_{2yr}$	Interest rate risk	$SD\left(Swaps_{10yr} - Swaps_{2yr}\right)$
Liquidity premium	$30USGov_{On} - 30USGov_{Off}$	Liquidity risk	$SD\left(30USGov_{On} - 30USGov_{Off}\right)$
Credit risk premium	$Swaps_{10yr} - TBills_{3Mo}$	Financing/hedging risk	$C_{K=S_t,T}^{S=Swaps-Tbill} = \max\left[0, S_T - K\right]$

Empirical Results – Part I

Results for Hedge Funds Engaged in Fixed Income Investing:

(2) $$return_{n,t} = \alpha + b_{1,t}\left[SWAPS30_2_t\right] + \xi_t \text{, where } \alpha = 0$$

The One-factor Model Results:

Figure III. One-Factor Model Results For Fixed Income Hedge Fund Return Sample

One-factor Regression ***SWAP30_2***	b_1	t-statistic
Preshock	-0.20	-2.90**
(SE)	(0.06927)	
Shock	-0.24	-3.77**
(SE)	(0.06493)	
Postshock	-0.04	-0.65
(SE)	(0.05818)	

Preshock: N=201, R^2=0.040, F=8.387, SE=0.03306; Shock: N=224, R^2=0.059, F=14.226, SE=0.05006; Postshock: N=296, R^2=0.000, F=0.072, SE=0. 02847

Empirical Results – Part I

Results for Hedge Funds Engaged in Fixed Income Investing:

(3) $return_{n,t} = \alpha + b_{1,t}[SWAPS30_2_t] + b_{2,t}[ON/OFF_30_t] + b_{3,t}[STRADDLE3M_10Y_t] + \eta_t$, where $\alpha = 0$

Three-factor Model Results:

Figure III-A. Full Sample of Fixed Income Hedge Funds

This table reports the variables and coefficients for regressions of monthly fixed income hedge fund returns (sorted according to preshock, shock and postshock months) versus contemporaneous mean standard deviations of spreads (*SWAPS30_2*; *ON/OFF_30*) and the one-month mean daily returns of straddles (*STRADDLE3M_10Y*). *SWAPS30_2* is the difference in rates between 30 year and 2 year interest rate swaps. *ON/OFF_30* is the difference in yield between on-the-run and off-the-run 30 year Treasury bonds. *STRADDLE3M_10Y* is the difference in rates between the 3 month treasury bill and 10 year swaps.

Three-factor Regression	b_1 ***SWAPS30_2***	**t-stat**	b_2 ***ON/OFF_30***	**t-stat**	b_3 ***STRADDLE3M_10Y***	**t-stat**
Preshock	-0.40	-4.16**	-0.30	-3.00**	0.02	0.27
(SE)	(0.09724)		(0.09979)		(0.07343)	
Shock	-0.11	-1.40	-0.21	-2.68**	-0.06	-0.77
(SE)	(0.07911)		(0.07677)		(0.07520)	
Postshock	0.06	0.86	0.07	1.00	0.21	2.81**
(SE)	(0.07235)		(0.06744)		(0.07658)	

Preshock: N=201, R^2=0.072, F=6.178, SE=0.03243; Shock: N=224, R^2=0.858, F=8.011, SE=0.04925; Postshock: N=296, R^2=0.018, F=2.769, SE=0.02819

Empirical Results – Part I (Cont'd)

How were fixed income funds sorted into more constrained versus less constrained funds?

More Constrained versus Less Constrained Funds:

Figure IV. Mean and Standard Deviation (SD) of Preshock Return for Groups of Funds by Cohort

SD of Preshock Returns	**Preshock Returns**		**Shock Returns**		**Postshock Returns**	
Groups	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Higher SD	0.8903	0.4135	-1.1874	6.7738	0.2615	3.5255
Lower SD	1.0367	0.0978	-0.6774	3.1167	0.9583	1.2239

How does RBL change in response to shocks?

Figure V. Estimates of Risk-Based Leverage and Volatility Exposure by Cohort and Fund Group

Groups	**Higher Preshock SD Group**		**Lower Preshock SD Group**	
Periods	*RBL*	*Vol Exposure*	*RBL*	*Vol Exposure*
Preshock	0.63	-0.03	1.31	0.23
Shock	0.31	-0.10	0.29	-0.07
Postshock	0.15	0.17	0.36	0.33

Empirical Results – Part I (Cont'd)

More Constrained versus Less Constrained Funds (cont'd):

Figure VI-A. Subgroup of Funds with Higher SD Preshock Return

This table reports the variables and coefficients for regressions of monthly returns for fixed income hedge fund with higher preshock standard deviation of returns (sorted according to preshock, shock and postshock months) versus contemporaneous mean standard deviations of spreads (*SWAPS30_2*; *ON/OFF_30*) and the one-month mean daily returns of straddles (*STRADDLE3M_10Y*).

Three-factor Regression	b_1 ***SWAPS30_2***	**t-stat**	b_2 ***ON/OFF_30***	**t-stat**	b_3 ***STRADDLE3M_10Y***	**t-stat**
Preshock	-0.37	-3.06**	-0.26	-2.10**	-0.03	-0.28
(SE)	(0.14832)		(0.15211)		(0.10734)	
Shock	-0.08	-0.70	-0.23	-1.89**	-0.10	-0.95
(SE)	(0.13641)		(0.13192)		(0.12431)	
Postshock	0.12	1.20	0.03	0.31	0.17	1.62
(SE)	(0.11486)		(0.09517)		(0.12143)	

Preshock: N=219, R^2=0.06, F=3.631, SE=0.01263; Shock: N=106, R^2=0.096, F=4.773, SE=0.03017; Postshock: N=128, R^2=0.13, F=7.536, SE=0.01444

Figure VI-B. Subgroup of Funds with Lower SD Preshock Return

This table reports the variables and coefficients for regressions of monthly returns for fixed income hedge fund with higher preshock standard deviation of returns (sorted according to preshock, shock and postshock months) versus contemporaneous mean standard deviations of spreads (SWAPS30_2; ON/OFF_30) and the one-month mean daily returns of straddles (STRADDLE3M_10Y).

Three-factor Regression	b_1 ***SWAPS30_2***	**t-stat**	b_2 ***ON/OFF_30***	**t-stat**	b_3 ***STRADDLE3M_10Y***	**t-stat**
Preshock	-0.73	-4.95**	-0.58	-3.83**	0.23	2.16**
(SE)	(0.122334408)		(0.12563)		(0.09525)	
Shock	-0.29	-2.14**	-0.14	-1.04	- 0.07	0.56
(SE)	(0.121813859)		(0.11984)		(0.10828)	
Postshock	-0.15	-1.31	0.21	2.22**	0.33	2.75**
(SE)	(0.09718113)		(0.09069)		(0.10328)	

Preshock: N=82, R^2=0.21, F=8.242, SE=0.04080; Shock: N=80, R^2=0.093, F=3.720, SE=0.06507; Postshock: N=168, R^2=0.001, F=0.955, SE=0.03526

Empirical Results – Part I (Cont'd)

Failed Fixed Income Arbitrage Hedge Fund Sample:

- Monthly returns reported in TASS "Graveyard File" for fourteen (14) hedge funds
- Returns for funds that liquidated during the sample period February 1997 through March 2000
- Monthly returns sorted into preshock, shock, and postshock months based on the standard deviation of underlying spreads for the month in question

Figure IV. Mean monthly returns for survivors and failures compared to TASS Fixed Income Arb return

Mean Mthly Returns	**Survivors**	**Fixed Income Arb Index**	**Failures**
Preshock	0.9501	0.9125	1.0200
Shock	-0.7924	-0.3914	-3.1305
Postshock	0.5628	0.5969	0. 2884

Empirical Results – Part I (Cont'd)

Out-of-Sample Test (cont'd):

Figure IV-A. The Change in *RBL* for Funds That Failed During the Sample Period

	Survivors		**Failures**	
	RBL	*Vol Exposure*	*RBL*	*Vol Exposure*
Preshock	0.7	--	0.3	--
Shock	0.2	--	0.8	+0.3
Postshock	--	+0.2	--	--

Empirical Results – Part I (Cont'd)

Out-of-Sample Test (cont'd):

Figure IV-B. Funds That Failed During the Sample Period

This table reports the variables and coefficients for regressions of monthly returns for fourteen (14) fixed income hedge funds that liquidated during the sample period (sorted according to preshock, shock and postshock months) versus contemporaneous mean standard deviations of spreads (*SWAPS30_2*; *ON/OFF_30*) and the one-month mean daily returns of straddles (*STRADDLE3M_10Y*).

Three-factor Regression	b_1 ***SWAPS30_2***	**t-stat**	b_2 ***ON/OFF_30***	**t-stat**	b_3 ***STRADDLE3M_10Y***	**t-stat**
Preshock	-0.12	-1.48	-0.26	-3.06**	0.08	0.93
(SE)	(0.08299)		(0.08622)		(0.08636)	
Shock	-0.26	-2.26**	-0.50	-4.27**	0.27	2.87**
(SE)	(0.11423)		(0.11608)		(0.09453)	
Postshock	0.08	0.64	0.18	1.33	0.21	1.33
(SE)	(0.13086)		(0.13766)		(0.15611)	

Preshock: N=141, R^2=0.08, F=4.186, SE=0.02115; Shock: N=95, R^2=0.38, F=18.407, SE=0.05625; Postshock: N=84, R^2=0.03, F=0.791, SE=0.04785

Empirical Design – Part II

Description of Data Sample for Broader Sample of Hedge Funds Engaged in Diverse Strategies:

- Monthly returns for a sample of 251 hedge funds that report returns in TASS for every month from February 1997 through March 2000

Variable Selection for a Regression Model on Broader Sample of Hedge Funds Engaged in Diverse Strategies:

- Monthly returns for seven asset indices: S&P500, US$INDEX, GSCI, RUSSELL, MSCIWRLD, MSCIEAFE, and LEHMAN COMP

- Monthly returns to three common trading strategies summarized from the sample of hedge fund returns using *principal components analysis*

- Monthly returns to three passive options portfolios constructed to be sensitive to changes in volatility (*STRADDLE*, *STRANGLE* (20% OTM), *COLLAR* (25% OTM))

Empirical Design – Part II (Cont'd)

Figure VI. *Average Returns of Hedge Funds by Strategy for the Sample Period*

	Average Returns		
Strategy	**Preshock**	**Shock**	**Postshock**
All Strategies	2.20	0.06	1.49
Convertible Arb	1.27	0.49	1.05
Equity Mkt Neutral	1.18	1.23	0.89
LongShrt Equity	3.23	2.88	2.63
Event Driven	1.76	0.18	1.11
Mng Futures	1.33	0.60	0.26
Global Macro	1.69	-1.24	1.57
Fixed Income Arb	0.91	-0.39	0.60
Short Sellers	-0.91	-0.70	-0.57

Source: TASS

Empirical Design – Part II (Cont'd)

For the broader sample, what is the basis for sorting monthly hedge fund returns in *Preshock*, *Shock*, and *Postshock* periods?

Daily stock returns as a proxy for asset mispricing (Richards, 1999)

- Ross (1976, 1977) – Arbitrage Pricing Theory (increased residual returns↔increased asset pricing errors)
- Campbell, Lettau, Malkiel and Xu (2000) – Idiosyncratic Risk of Stocks
- Erb, Campbell and Viskanta (1995) – Correlation of Global Equity and Fixed Income Markets

Figure VII. *Shock Periods for the Broader Sample of Funds*

Event	**Dates**
Asia Crisis	8/97, 9/97, 10/97
Russia Default, LTCM Crisis	8/98, 9/98, 10/98, 12/98
Brazil Crisis	1/99
Swap curve "events" (the repurchase of 30-Year US Treasuries)	5/99, 7/99, 10/99
NASDAQ (Internet Stock) Bubble	1/00

Empirical Design – Part II (Cont'd)

In the second sample, how were shock months identified?

The thirty (30) stocks from Dow Jones Industrial Average (DJIA). The residual variance estimates $\hat{r}_{g,t}^{1}$ and $\hat{r}_{g,t}^{2}$ are based on 1344 observations, during the period January 1995 through April 2000, for each of the 30 stocks comprising the DJIA.

Two of the three methods employed by Richards (1999) to estimate "$\hat{r}_{g,t}$" (the residual variance exhibited by specific stocks in the DJIA relative to the remaining stocks in the index):

(A) The square root of the sum of squared differences over the period:

$$\hat{r}_{g,t}^{1} = \Sigma(r_{g,t} - \Sigma\frac{1}{G}[r_{g,t}])^2$$

($\hat{r}_{g,t}^{1}$ spuriously larger during periods with large movements in the common component of returns)

(B) The residual in the OLS regression: $r_{g,t} = a_{0,t} + a_{2,t}\, r_{2,t} + \ldots + a_{n-1,t}\, r_{n-1,t} + e_{g,t}$

$$\hat{r}_{g,t}^{2} = e_{g,t} = r_{g,t} - \hat{a}_{0,t} - \Sigma_{h=1}^{n-1}\, \hat{a}_{h,t}\, r_{h,t}$$

($\hat{r}_{g,t}^{2}$ less susceptible than $\hat{r}_{g,t}^{1}$ to the missing variables problem; it does not account for time-varying factor coefficients in asset returns)

Principal Components Analysis

Defn. – *Principal components*: *l* uncorrelated, linear combinations of *p* random variables $\mathbf{X}' = [X_{1,} X_2, X_3, ... X_p]$, that are fewer in number (*l*<*p*, where *p* equals the number of hedge funds in the sample), and more parsimoniously explain the variance-covariance structure of the original variables (Johnson and Wichern, 1998).

Achieved by maximizing sample variation, i.e. choosing the *l* out of $\mathbf{Y}' = [Y_{1,} Y_2, Y_3, ... Y_p]$ linear combinations with maximum variance, such that the $Var(Y_1) > Var(Y_2) > ... > Var(Y_p)$.

Computing principal components depends solely on $\boldsymbol{\Sigma}$, the covariance matrix (or $\boldsymbol{\rho}$ the correlation matrix in the case of standardized returns) of $\mathbf{X}' = [X_{1,} X_2, X_3, ... X_p]$ with eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq ... \geq \lambda_p \geq 0$. Hence, MVN unnecessary…

The linear combination $\mathbf{a}'\mathbf{X} = a_1 X_1 + a_2 X_2 + a_3 X_3 + ... + a_p X_p$ has a mean equal to $E[\mathbf{a}'\mathbf{X}] = \mathbf{a}'\boldsymbol{\mu}$, and variance equal to $Var(\mathbf{a}'\mathbf{X}) = \mathbf{a}'\boldsymbol{\Sigma}\mathbf{a}$, where $\boldsymbol{\mu} = E[\mathbf{X}]$ and $\boldsymbol{\Sigma} = Cov(\mathbf{X})$.

The multivariate vector of linear combinations $\mathbf{Y} = \mathbf{AX}$ possess $\boldsymbol{\mu}_Y = E[\mathbf{Y}] = E[\mathbf{AX}] = A\boldsymbol{\mu}_Y$, and $\boldsymbol{\Sigma}_Y = Cov(\mathbf{Y}) = Cov(\mathbf{AX}) = \mathbf{A}\Sigma_{\mathbf{Y}}\mathbf{A}'$.

We obtain $Var(\mathbf{Y}) = \mathbf{a}_k'\Sigma\,\mathbf{a}_k$ and $Cov(\mathbf{Y}_k, \mathbf{Y}_l) = \mathbf{a}_k'\boldsymbol{\Sigma}\mathbf{a}_l$, where $k \neq l$ and $k, l = 1,2,3,..., p$.

The procedure repeats for *l*<*p* principal components, where the *l*th principal component is the linear combination $\mathbf{a}_1'\mathbf{X}$ that maximizes $Var(\mathbf{a}_1'\mathbf{X})$ subject to $\mathbf{a}_1'\mathbf{a}_1 = 1$ and $Cov(\mathbf{a}_2'\mathbf{X}, \mathbf{a}_2\mathbf{X}) = 0$.

Volatility Factors

How were option values computed in order to the construct proxies for volatility factors?

Values are calculated for 30-day *European* options, using daily prices and implied volatilities for the S&P500 index, during the period from January 1997 to March 2000.

$$return_{option,month"t"} = \Sigma_{Daily(t-1,t)} \left[return_{tradingdays(\tau-1,\tau)} \right] \times OptionValue_{monthend,t-1}$$

The value for an option position at month-end is equivalent to the cumulative sum of the daily returns during that month multiplied by the value of the position at the beginning of the month.

VOLFACTORS **(Option-Based Proxies)**	**Passive Portfolio Position**
ATMCALL	$C^S_{K=S_t,T} = \max[0, S_T - K]$ [1]
ATMPUT	$P^S_{K=S_t,T} = \max[K - S_T, 0]$ [2]
STRADDLE	$C^S_{K=S_t,T} + P^S_{K=S_t,T}$
STRANGLE	$C^S_{K=120\%S_t,T} + P^S_{K=80\%S_t,T}$
COLLAR	$-C^S_{K=125\%S_t,T} + P^S_{K=75\%S_t,T}$

[1] Where S_T is the underlying stock price at maturity and K is the "strike", i.e., the *exercise price* of the option. In the case of ATM options, $K=S_{t=0}$ the price of the underlying at the initial settlement date of the option.

[2] In practice, the principle of perfect Put-Call parity does not hold. The value of the ATM put is only approximately equivalent to the value of the ATM call, due to the asymmetry of the underlying stock process.

Empirical Design – Part II (Cont'd)

The FH1997 regression

(4) $return_{n,t} = \alpha + \Sigma_i \left[b_i Assets_{i,t} \right] + \Sigma_f \left[b_f Strategies_{f,t} \right] + u_t$

where,

$return_{n,t}$ = the return for month t of the nth fund in the sample of 251 hedge funds

$ASSETS_{i,t}$ = $i = 7$ proxies for one-month "buy-and-hold" asset returns ($i = 7$)

$STRATEGIES_{i,t}$ = f proxies for one-month returns to hedge fund trading strategies ($f = 3$)

and, $t = 39$ months in the sample from January 1997 to March 2000, sorted as *preshock*, *shock*, and *postshock*

Empirical Design – Part II (Cont'd)

Multi-factor regression for returns of of hedge funds engaged in diverse strategies
Extends FH1997:

(5) $$return_{n,t} = \alpha + \Sigma_i \left[b_i Assets_{i,t} \right] + \Sigma_j \left[b_j Strategies_{j,t} \right] + \Sigma_k \left[b_k VolFactors_{k,t} \right] + \varepsilon_t$$

where,

$return_{n,t}$ = the return for month t of the nth fund in the sample of 251 hedge funds

$ASSETS_{i,t}$ = i proxies for one-month "buy-and-hold" asset returns ($i = 7$)

$STRATEGIES_{i,t}$ = f proxies for one-month returns to hedge fund trading strategies ($f = 3$)

VOLFACTORS = factors of one-month returns for volatility sensitive option portfolios (k = 3)

and, $t = 39$ months in the sample from January 1997 to March 2000, sorted as *preshock*, *shock*, and *postshock*

Empirical Results Part II (Cont'd)

Figure VIII. The Change in ***RBL*** for the Full Sample of Hedge Funds Engaged in Diverse Strategies

(at the 10% significance level)	***RBL***	**Net Volatility Exposure**
Preshock	34.02	0.00
Shock	16.24	13.28
Postshock	0.00	0.00

Figure IX. The Change in Strategy Exposure for the Full Sample of Hedge Funds

(t-stat in parentheses)	***STRATEGY1***	***STRATEGY2***	***STRATEGY3***
Preshock	-27.25 (-1.71)	-9.10 (-1.34)	0.15 (2.05)
Shock	13.17 (1.80)	2.77 (2.23)	-0.16 (-1.19)
Postshock	-1.75 (0.00)	1.40 (0.00)	0.12 (0.00)

STRGY1 = Proxy for common "factor" returns correlated with convertible arbitrage and US equity hedge strategies
STRGY2 = Proxy for common "factor" returns correlated with risk arbitrage and European equity hedge strategies
STRGY3 = Proxy for common "factor" returns correlated with managed futures strategies

Limitations

Reduced statistical significant and misestimation of absolute (vs. relative) *RBL*

- Multicollinearity (Kennedy 1993)
- Heteroskedasticity and Non-stationarity

Further Research

1. Within the samples, what strategies exhibit greater sensitivity of returns to increasing volatility?

2. Do the characteristics of the individual funds comprising the sample confirm the findings? How might these characteristics affect the results?

3. For the broader, more general sample of hedge funds, can we observe similar differences in their sensitivity to volatility increases?

4. How influential is period during the 1998 LTCM collapse on the sample results?

5. How persistent is mispricing during those shock periods, when hedge funds exhibit reduced exposure?

Additional Slides

Presentation Outline

1. Research Question – empirically documenting the behavior of hedge funds as "professional arbitrageurs"

2. Contribution – development and application of an econometric methodology for a problematic empirical analysis

3. Relevance of Question & Contribution
   - The Role and Implications of Arbitrage in Price Efficiency
   - Comparative Implications of Classical Arbitrage versus Limits of Arbitrage
   - Empirical Implications of Shleifer and Vishny's Model of the Limits of Arbitrage
   - Anecdotal Evidence from the Hedge Fund Industry on the Limits of Arbitrage
   - Challenges to Empirically Testing the Implications of Shleifer and Vishny's Model
   - Changes in risk-based leverage as Metric for Changes in Hedge Fund Investment
   - Estimating Changes in risk-based leverage Based on Available Data on Hedge Fund Returns

4. Hypotheses related to observed changes in risk-based leverage and returns, conditioned on changes in volatility

5. Empirical Design
   E. Two Sets of Sample Data: Fixed income hedge funds; Diverse hedge fund strategies
   F. Variable Selection
   G. Regression Specification(s): Single-Factor and Multiple-Factor Regressions

6. Results of the first sample regressions: One-factor and Three-factor regressions on the returns of hedge fund engaged exclusively in fixed income investment strategies

7. Results of the second sample regressions: Multiple-factor regressions on the returns of a broad class of hedge funds engaged in diverse strategies

The Role of Arbitrage in the Economic Literature: Maintaining Pricing Efficiency

Classical Arbitrage ("*CA*"): Arbitrageurs are more aggressive when volatility increases (higher expected returns)

Grossman and Miller (1988) – Liquidity Risk and Market Structure

De Long, J.B., A. Shleifer, L. Summers, R. Waldmann (1990) – Noise Trader Risk

Campbell and Kyle (1993) – Noise Trading and Stock Prices

– vs. –

Limits of Arbitrage ("*LoA*"): Arbitrageurs are less aggressive when volatility increases despite the profit incentive

Shleifer and Vishny (1997) – Model of Arbitrage Limited by Shocks to Asset Prices ("SV1997")

The notion that practically speaking, since in the short-run, arbitrage is risky, extreme volatility acts as a disincentive for arbitrageurs

The General Implication for (Short-Run) Price Efficiency:

The existence of *LoA* might influence the rate and degree to which prices adjust during volatile markets

The Primary Empirical Result of SV1997

Although market volatility increases the potential for mispricing and thus higher expected returns, hedge fund performance (and subsequent access to capital) being adversely affected by price shocks result in hedge funds *reducing*, rather than increasing investment exposure during periods of increased market volatility.

Recent theoretical research explores LoA in the context of various capital constraints to show that constraints, such as margin requirements and bankruptcy costs, limit the trading activity (i.e. investment exposure) of hedge funds.

Liu and Longstaff (2000) – Risk of Interim Losses Lead to Longer-Term Underinvestment by Arbitrageurs

Vayanos and Gromb (2000) – The Effects of Margin Requirements as a Financial Constraint on Arbitrageurs

Xiong (1999) – How Wealth Effects Relate to Financial Contagion (Arbitrageurs as "Convergence Traders")

Yuan (1999) – The Relationship between Asymmetric Price Movements and Borrowing Constraints

Anecdotal Evidence Supporting LoA and SV1997

Hedge funds fit the description of *professional arbitrageurs* as described by SV1997:

Hedge funds are believed to play an important role in aligning prices by employing leverage, short-selling and derivatives (e.g. options) to exploit capture excess returns from asset mispricing. To do so hedge funds invest as agents on behalf of principals who provide capital.

The hedge fund sector has grown over $150 billion of AUM (assets under management) over the past year to $750 billion and is expected to exceed $1 Trillion of AUM within a few years.

Anecdotal evidence during extreme market events affirm that severe volatility shocks seem to trigger the simultaneous unwinding of their positions

Dunbar (1999); Lowenstein (2000) – The LTCM Collapse of 1998

Getmansky, Lo and Makarov (2003) – Serial Correlation of Hedge Fund Returns

Goldman (2003) – Hedge Fund Risk Assessment, Survival, and Leverage

Related Empirical Research

Fung and Hsieh ("FH1997") – employ multi-factor regression to compare, across market rallies and declines, changes in the investment leverage (relative to long-run averages):

- Compare the leverage of hedge funds engaged in diverse strategies to that of mutual funds
- Find investment leverage on average to be considerably higher and far more variable for hedge funds relative to mutual funds

Brown, Goetzmann and Park ("BGP2001") – examine the cross-sectional and time-varying changes in the risk preferences of hedge funds in general, in order to draw inference regarding survival and competition among hedge fund managers

- Compare changes in the excess returns and standard deviations of returns comprising their Sharpe ratios
- Find that managers with mediocre or worse performance increase the variance of their returns, versus a reduction in variance by managers with the best performance.

However, neither paper focuses specifically on changes in risk-based leverage conditioned on changes in underlying spread volatility, nor do either of these papers employ options-based proxies to test the sensitivity to increased volatility of the hedge fund returns within the sample.

For example:

Neither FH1997, nor BGP2001 distinguish between periods that exhibit greater versus lesser volatility, nor do they classify groups of funds according to their relative sensitivity to increases in volatility.

BGP2001 also does not account for differences variance of returns across diverse hedge fund strategies, or the sensitivity of different strategies to volatile market conditions.

Related Empirical Research (cont'd)

Prior research on options-based performance evaluation of managers related to the use of an options-based proxy for measuring the volatility-sensitivity of returns:

Agarwal and Naik (2000)*

Bookstaber and Clark (1984)**

Fung and Hsieh (1998b, 2001)*

Merton, Scholes and Gladstein (1978, 1982)**

*Applied to hedge fund returns
**Applied to mutual fund returns

Multifactor Model

The multi-factor regression framework, based on the APT (Ross, 1976), introduced by Sharpe (1992), and applied by FH1997

Let R_t = return on a portfolio of assets in period *t*,

x_{jt} = the weight of asset *j* within the portfolio during period *t* (from $t-1$ to *t*), and

r_{jt} is the return of the asset *j* within the portfolio during period *t,*

$j = 0,...,J$, and $\sum_j$ denotes the summation operator overall values of *j*.

For convenience, the $j = 0$ asset is the risk-free asset. Hence,

$R_t = \sum_j x_{jt} r_{jt}$ and $r_{jt} = \sum_k \lambda_{jk} F_{kt} + \varepsilon_{jt}$.

There are *K* systematic factors, $F_{kt}, k = 1,...,K$; λ is the factor loading; and ε represents are the idiosyncratic returns.

By assuming the systematic factors to be exogenously specified, one can interpret the factors as *asset classes*, following Sharpe (1992). Thus, using the factor model, the portfolio returns can be rewritten as:

$R_t = \sum_j \omega_{kt} F_{kt} + e_t$, where, $\omega_{kt} = \sum_j x_{jt} \lambda_{jk}$ and $e_t = \sum_j x_{jt} \varepsilon_{jt}$.

Thus, $R_t = \alpha + \sum_k b_k F_{kt} + u_t$

Appendix Section 5. Regression Coefficients for the Diverse Hedge Fund Sample

This table displays the coefficients from the regression of the full sample of monthly returns during the period from January 1997 through March 2000, as reported in the TASS database for 251 hedge funds engaged in diverse strategies. The returns are sorted according to preshock, shock, and postshock months, and regressed against contemporaneous returns for the following variables *ASSETS* (seven (7) indices representing returns of US and non-US stocks; currencies; commodities; and government, mortage-/asset-backed, and corporate bonds), *STRATEGIES* (three (3) dominant trading strategies underlying the sample identified using principal components analysis), and *VOLEXPOSURE* (three (3) options-based proxies for sensitivities of returns to changes in volatility and volatility skews).

	S&P500	***US$INDEX***	***GSCI***	***MSWRLD***	***RUSSELL***	***MSEAFE***	***LEHCOM***	***STRGY1***	***STRGY2***	***STRGY3***	***STRADL***	***STRNGL***	***COLLAR***
Preshock	--	-0.91	-0.44	2.45	1.32	-0.56	-2.86	-27.25	-9.10	0.15	1.04	--	--
n = 2730		(-1.62)	(-1.54)	(2.07)**	(1.90)*	(-1.52)	(-1.94)*	(-1.71)*	(-1.34)	(2.05)**	(1.53)		
R^2=0.163													
Shock	3.45	-1.66	-0.96	-4.93	0.29	0.64	-0.93	13.17	2.77	-0.16	-74.13	77.25	10.16
n = 3663	(1.47)	(-1.22)	(-1.61)	(-1.40)	(2.46)**	(0.96)	(-1.18)	(1.80)*	(2.23)**	(-1.19)	(-1.72)*	(1.72)*	(1.71)*
R^2=0.147													
Postshock	0.29	0.06	0.18	-0.17	0.47	-0.42	0.08	-1.75	1.40	0.12	0.38	0.58	0.34
n = 3144	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
R^2=0.139													

Fund Return =Monthly hedge fund returns sorted according to preshock, shock and postshock months
S&P500 = Monthly returns for the S&P500 Index
US$INDEX = Monthly percentage change for the US Dollar Trade-Weighted Index
GSCI = Monthly percentage change for Goldman Sachs Commodity Index
MSWRLD = Monthly returns for for the Morgan Stanley World Equity Index (USD)
RUSSELL = Monthly returns for the Russell 2000 Index of small stocks
MSEAFE = Monthly returns for the Morgan Stanley Equity Index Europe, *Australasia* and Far East, Developed Markets only (USD)
LEHCOM= Monthly percentage change for the Lehman Composite Bond Index
STRGY1 = Proxy for common "factor" returns correlated with convertible arbitrage and US equity hedge strategies as defined and reported by TASS
STRGY2 = Proxy for common "factor" returns correlated with risk arbitrage and European equity hedge strategies as defined and reported by TASS
STRGY3 = Proxy for common "factor" returns correlated with managed futures strategies as defined and reported by TASS
STRDL = *Long position* of 30-day at-the-money European puts and calls on the S&P500 Index
STRNGL = *Long position* of 30-day 20% out-of-the-money European puts and calls on the S&P500 Index
COLLAR = *Net long position* of 30-day *long* 25% out-of-the-money European puts and *short* 25% out-of-the-money European calls on the S&P500 Index