Price Pressure and Price Discovery in the Term Structure of Interest Rates*

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ABSTRACT

We study the price pressure and price discovery effects in the U.S. Treasury market by using a term structure model. Our model decomposes yield curve shifts into two components: a virtually permanent change related to order flow and a transitory, price pressure effect due to dealer inventories. We find strong evidence that net dealer Treasury inventories has impact on the yield curve. Cash Treasury instruments in inventory have a larger impact on yields than futures contracts, suggesting that cash and futures inventories are not perfect substitutes. Price discovery in the level of interest rates is most strongly linked to non-dealer order flow in the 10-year futures contract, while price discovery in the slope of the curve is linked to order flow in the 10-year futures and the 5-year cash market.

JEL classification: G10, G120, G140

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I. Introduction

Dealers typically provide liquidity for multiple instruments. Holding inventory in one of these instruments can influence dealer behavior and pricing in all of the relevant markets. In addition, order flow and price discovery in one market is likely to feed through to related markets. An important special case of this type of market integration is the U.S. Treasury market. Dealers make markets by using cash and derivatives instruments across the term structure of U.S. interest rates. As Grossman and Miller (1988); Stoll (1978) suggest, Treasury dealers should demand price concessions to hold risky inventory. At the same time, order flow in a specific Treasury security impacts the entire yield curve.

Our goal in this paper is to analyze the effects of dealer inventory and order flow on Treasury yields by using a term structure model. Our strategy is to modify the term structure model to allow for mispricing to depend on dealer inventory and the factors governing the yield curve to be impacted by order flow. The resulting specification is flexible enough to estimate the price pressure and price discovery effects of different securities across different maturities. We use this flexibility to investigate the relative importance of Treasury cash and futures markets across different maturities for price discovery and liquidity provision.

We build on the dynamic Nelson-Siegel term structure factor models specified in Diebold, Rudebusch and Aruoba (2006) and related papers such as Diebold and Li (2006) and Christensen, Diebold and Rudebusch (2011). We characterize the dynamics of the term structure with three latent factors, and we further allow market yields to depend on the level of dealer inventory. Inventory, in this context, consists of dealer positions in both Treasury cash and futures across various maturities. Finally, we allow innovations in latent factors to be correlated with net non-dealer order flow of specific Treasury instruments. We, therefore, characterize price discovery taking place at the factor level, rather than occurring at the level of specific instruments, maturities, or markets. This modeling choice reflects the integration of the U.S. Treasury market across different maturities and instruments that are all impacted by the same factors.

We have three main findings. First, we find a statistically significant effect of dealer inventory on yields of Treasury securities with a similar maturity. Net positive (negative) dealer inventories are associated with higher (lower) market yields. This finding is consistent with dealers providing liquidity to the market in return for compensation through price concessions: a "price pressure" effect. The net effect on yields varies across the term structure and over time. For example, we estimate that dealers were typically short Treasury exposure during the 2001-2013 period, and we conclude that this decreased 10-year Treasury yield by nearly 5 basis points, on average. Second, we find evidence that long-dated interest rate exposure via cash instruments is associated with a larger inventory effect on yields than exposure via futures, indicating that cash and futures are not perfect substitutes in a dealer book. Third, our model accommodates a price discovery channel by linking non-dealer order flow to fundamental moves in the yield curve. Consistent with this channel, we find a significant link between order flow and latent factors that describe bond yield changes. Specifically, the links are strongest between order flow in the 5-year cash Treasury and movements in the front-end of the curve (through the slope factor) and between order flow in the 10-year Treasury future and movements in the back-end of the curve (through the level factor and the slope factor).

Arguably, the most challenging problem for identifying the price pressure effect is to decompose price changes into temporary and permanent price changes. Permanent price changes represent changes in fundamental asset values while temporary price changes could reflect liquidity conditions. Therefore, this decomposition is crucial because the price pressure effect is only related to the temporary component of price changes. In a state-space model estimation, Kalman smoother enables this decomposition and allows for analyzing the temporary component of price changes. Furthermore, state-space models have the advantage of estimating all model parameters in one step, improving the model fit. In fact, state-space models are already commonly used in modeling the term structure of Treasury yields (for example, Christensen, Diebold and Rudebusch (2011); Diebold, Rudebusch and Aruoba (2006)). These models can provide estimates of efficient Treasury yields, which is necessary for decomposing observed yields into fundamental and non-fundamental components.

We define price pressure as the deviation in observed price from the efficient price, that is attributable to the compensation required by intermediaries in order to hold risky inventory. As Stoll (1978) argued, this holding cost of intermediaries can also be interpreted as a measure of market liquidity. We model price discovery at the factor level, as opposed to the instrument level. We specify the transition equations for the latent factors to include non-dealer order flow, which is computed from observable data on cash and futures.

The core idea is that observed Treasury yields deviate from efficient yields due to dealer inventory and idiosyncratic noise. The efficient yields are given by a factor structure, as in Diebold, Rudebusch and Aruoba (2006), where innovations in latent factors are correlated with order flow. The strategy is to impose structure across maturities via the factor model in order to identify the price discovery process and the inventory effect separately.

Our major contribution is to study the price pressure and price discovery effects by utilizing state space models from the market microstructure literature and factor models from the term structure literature. We use insights from the microstructure field to model price discovery and inventory effects, and we use insights from the term structure literature to identify and control for common factors impacting yields. Hence, we are able to model the relevant properties of interest rates jointly, rather than simply presenting independent results from different maturities.

The rest of the paper is organized as follows. Section II reviews the related literature. Section III introduces our modeling approach. Section IV gives the details of our data and summary statistics. Section V provides the estimation results of the state-space model for the U.S. Treasury market. Section VI concludes.

II. Related Literature

This paper brings together key insights from four distinct segments of the finance literature. One segment of the literature emphasizes the inventory control process of market makers in determining price dynamics that layer on top of "fundamental" price changes. A second segment of the literature emphasizes the importance of bond supply and demand factors, unrelated to traditional macroeconomic factors, in determining the shape of the term structure. A third segment focuses on the dynamics of the price discovery process, often linking it to customer order flow. A fourth segment of the literature emphases the common factor dynamics across the yield curve. Next, we place our paper into the context of these research topics.

Researchers have exploited position data of intermediaries in order to test predictions of theories focused on inventory control as a determinant of price dynamics. Models such as those by Stoll (1978) and Grossman and Miller (1988) suggest that risk-averse liquidity providers expect to be compensated for holding risky inventory. Madhavan and Smidt (1991, 1993) and Hasbrouck and Sofianos (1993) provide evidence for intraday mean reversion in inventory of specialists on the New York Stock Exchange, as predicted by the theory. Hendershott and Seasholes (2007) test longer-term predictions of liquidity provision by market makers who maintain inventories. They find supporting evidence that specialist inventory levels predict future return reversals. Muravyev (2016) concludes that inventory risk faced by option market makers has a first-order effect on equity option prices. Naik and Yadav (2003) find that U.K. bond dealers use futures markets to manage the systematic risk of cash bond portfolios, but they do not completely eliminate this risk. Fleming and Rosenberg (2008) similarly conclude that dealers use futures to manage cash bond risk, and they find evidence that dealer inventory risk is priced for a brief period after Treasury auctions.

There is also a literature linking bond yields to aggregate bond supply and demand factors and arbitrage activity. There is empirical evidence that shocks to clientele demand and bond supply have explanatory power for Treasury yield curve changes, beyond that of standard yield curve factors or macroeconomic factors such as expected short-term interest rates and inflation.Greenwood and Vayanos (2010) present anecdotal evidence in support of this idea, and Vayanos and Vila (2009) and Kaminska, Vayanos and Zinna (2011) examine "Preferred Habitat" models of U.S. Treasury securities. In these papers, the term structure is determined by the interaction of investor clienteles with preferences for specific maturities of bonds (e.g., pension funds) and risk averse arbitrageurs who absorb their demands. Greenwood and Vayanos (2014) build on this model in an empirical examination relating the supply and maturity structure of Treasury securities to yields across the term structure. Krishnamurthy and Vissing-Jorgensen (2012) conclude that the supply of Treasury debt held by the public affects various yield spreads. Hamilton and Wu (2012) provide evidence that the maturity structure of all publicly held Treasury debt matters for the term structure. Li and Wei (2013) and others find significant, measurable effects of the Federal Reserve's Large-Scale Asset Purchase Program on yields. Hu, Pan and Wang (2013) link a non-fundamental component of Treasury yields to arbitrage activity across Treasury securities.

A third segment of the literature focused on price discovery in the U.S. Treasury market suggests that a significant amount of variation in yields is related to customer order flow in both cash and futures markets. Brandt and Kavajecz (2004) find that cash bond order flow explains over a quarter of the day-to-day variation in yields on non-macroeconomic announcement days, and they conclude that inventory effects play an immaterial role in the price dynamics. Pasquariello and Vega (2007) conclude that the impact of order flow varies over time depending on the underlying market environment. Brandt, Kavajecz and Underwood (2007) find that the order flow impact for cash bonds appears to be stronger at the front of the curve (e.g., 2-Year and 5-Year Notes), whereas the order flow impact for futures is stronger at the long end of the curve (e.g., 10-Year Notes and Bonds). This appears consistent with the finding by Mizrach and Neely (2008) that more price discovery takes place in cash markets in the short end of the curve but that, at the long end of the curve, more price discovery takes place in futures.

As noted by Muravyev (2016) and Hendershott and Menkveld (2014), separately identifying inventory effects from asymmetric information effects and price discovery is challenging, especially with intraday data. Both of these studies find that inventory imbalances have price effects that often last over multiple days, although a common assumption is that intraday price changes due to information are largely permanent but that inventory effects dissipate quickly within the day. These authors further emphasize that order flow and price changes are endogenously determined and that simple OLS regressions of price changes on order flow may produce biased results. We follow Hendershott and Menkveld (2014) and estimate a state-space model that decomposes yield changes into two components: a permanent change related to order flow and a transitory, price pressure effect due to dealer inventories.

We follow the literature related to Diebold, Rudebusch and Aruoba (2006), who model yield curve dynamics with a three-factor term structure model based on the Nelson and Siegel (1987) characterization. The latent factors are interpreted as level, slope, and curvature. These parsimonious models provide consistent descriptions of yield changes across the term structure and allow yields to interact with other variables. Whereas Diebold, Rudebusch and Aruoba (2006) allow the latent factors to interact with observable macroeconomic factors such as real activity and inflation, we augment the latent factors with observed order flow and dealer inventory data from the cash and futures markets. Deviations of observed Treasury yields from these true Treasury yields, due to dealer inventories, can provide a measure for the price pressure effect. More specifically, when Treasury dealers hold long (short) inventory, observed yields are expected to be higher (lower) than true yields in order to compensate them for taking on risky inventory. In our empirical specification, we augment a simple Nelson-Siegel term structure model with Treasury dealer inventories to allow for a price pressure effect. We measure inventories in terms of $DV01$ (Dollar value of a basis point). This choice produces parameter estimates that are expressed as an intuitive measure of Treasury market liquidity: the compensation liquidity providers charge per unit of unhedged risk exposure.

III. Modeling Approach

The intuition behind the model is that one can decompose yield changes into two components: (1) yield changes that reflect "fundamentals" or "information" and (2) temporary yield changes due to other factors, such as dealer inventory. The permanent price impact of a trade reveals the information content of a trade. The temporary price impact of a trade would be related to the compensation of liquidity providers for holding risky inventory. After liquidity providers increase (decrease) their inventory positions, prices are expected to increase (decrease) and reverse the temporary price impact. In other words, the price pressure due to the intermediary's inventory holding causes a temporary deviation from the true price. In practice, it is not straightforward to compute this temporary price deviation because one has to take a stance on what the true price is. For example, Hendershott and Menkveld (2014) uses a one-factor state-space model to decompose stock prices into fundamental and non-fundamental prices. In our application, we rely on the three-factor model of the term structure to generate the true, or fair, value of yields and relate yield deviations from fair value to dealer inventory. In extensions of the basic model, we allow innovations in the latent factors to be correlated with observed order flow surprises, thus linking price discovery and order flow.

We begin the description of the model by defining the observation equation of the state space model. On a given date t, observed bond yields reflect true bond yields plus an error term:

$$\widehat{y_t}(\tau) = y_t(\tau) + v_t(\tau). \tag{1}$$

The variable $\widehat{y_t}(\tau)$ represents the observed yield on a τ-maturity bond at time t, $y_t(\tau)$ is the efficient, or fair value, yield on a τ-maturity bond at time t, and $v_t(\tau)$ is a stationary pricing error. Both terms on the right hand side of (1) are latent processes.

We allow the stationary pricing error $v_t(\tau)$ to depend on the time t dealer inventory of bonds maturing at time τ. We consider the case where inventory is observed for maturities $\tau = \tau_1, \tau_2, ..., \tau_N$. The pricing error evolves according to the equation:

$$v_t(\tau) = \pi_{\tau,\tau} I_t(\tau) + \epsilon_t(\tau), \tag{2}$$

where $I_t(\tau)$ is the time t dealer inventory of the bond maturing at time τ and ϵ_t is idiosyncratic noise. The level of inventory affects the error through the parameter $\pi_{i,j}$, which is the coefficient linking the yield of of the bond maturing at time i with the inventory of bonds maturing at time j. If dealers are compensated for holding risky inventory of a particular maturity, we expect the prices of those bonds to be temporarily lower than otherwise when the inventory is positive. In our benchmark specification, we assume that inventory affects only the bonds with the same maturity, or that $\pi_{i,j} = 0$ for $i \neq j$. Because yields move inversely to prices, we expect to find that $\pi_{i,j} >= 0$ for $i = j$, suggesting that positive inventories are related to lower prices and higher market yields.

The efficient yield for a given maturity τ is determined via the following equation:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau} - e^{-\delta\tau}\right), \tag{3}$$

where the latent factors β_{1t}, β_{2t}, and β_{3t} are interpreted as time-varying level, slope, and curvature factors. The terms multiplying them are the factor loadings for a given maturity. Following Diebold, Rudebusch and Aruoba (2006), we fix the parameter δ at the value 0.0609.

Let $OF_t(\tau)$ be the time t non-dealer order flow for bonds maturing at time τ, observed for maturities $\tau = \tau_1, \tau_2, ..., \tau_N$. For example, if dealer clients are net buyers of bonds, order flow is positive. Then define the vector $OF_t = (OF_t(\tau_1), OF_t(\tau_2), ..., OF_t(\tau_N))'$. This allows us to specify the transition equation governing the dynamics of the three dimensional state vector as

$$(\beta_t - \mu) = \Theta(\beta_{t-1} - \mu) + \Lambda OF_t + \omega_t, \tag{4}$$

where Θ is a 3×3 diagonal matrix determining the autoregressive properties of the state vector and ω_t is idiosyncratic noise. The matrix Λ is a $3 \times N$ array that allows order flow to impact each of the level, slope, and curvature factors. This specification lets order flow at date t have a one-time, permanent impact on each factor, allowing for order flow to drive price discovery. μ is a 3×3 vector of mean values for the factors.

We now collect the specifications in equations (1), (2), and (3) and combine them using obvious vector notation:

$$\widehat{y_t} = \Gamma\beta_t + \Pi I_t + \epsilon_t. \tag{5}$$

The vector formulation in (5) concisely presents the observation equation representing observed cash market Treasury yields for N observed maturities as the sum of three components. The first component is a deterministic factor loading matrix Γ times a vector of latent level, slope, and curvature factors, representing common movements across instruments with differing maturies. The second component is an $N \times 1$ vector of non-dealer order flow observations pre-multiplied by a diagonal $N \times N$ matrix of "price pressure" coefficients; this component generates idiosyncratic deviations in yield, associated with dealer inventory at those points on the curve, from "fair value". The final component is idiosyncratic noise. Finally, we collect distributional assumptions:

$$\begin{pmatrix} \omega_t \\ \epsilon_t \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right]. \tag{6}$$

In our benchmark formulation, we assume that both the covariance matrix Q and the covariance matrix H are diagonal.

The dynamic movements of the latent factors are governed by a first order autoregressive process, augmented by contemporaneous values of the non-dealer order flow OF_t, a four-dimensional vector with elements corresponding to the non-dealer position changes for each of the four observed maturity groupings. That is, the first element corresponds to the non-dealer position change in the 2-year Note, and so forth.

We assume that non-dealer traders demand liquidity and incorporate information into prices through trading (although we do not assume that all price changes need to be linked to order flow). The trades of liquidity demanding traders are modeled to impact the latent factors directly. An alternative would be to model yields and order flow jointly, parallel to Diebold, Rudebusch and Aruoba (2006), who model yields and traditional macroeconomic factors jointly. Their methodology allows for various hypothesis tests regarding the relation among variables and allows for exercises such as impulse response analyses. In our application, however, our focus on allowing surprise order flow to have a contemporaneous impact on yields and on allowing the level of inventory to affect yields, leads us to prefer our formulation.

IV. Data and Summary Statistics

We use three main types of data for the analysis: zero-coupon Treasury yield data computed as in Gürkaynak, Sack and Wright (2007), aggregated dealer cash positions in Treasury securities, and aggregated dealer futures and options positions on Treasury securities. The final sample of data is weekly, as of the close of business on Wednesdays, and spans the period July 5, 2001 to April 18, 2018.

The position data for cash instruments is reported to the Federal Reserve Bank of New York, as of the close of business each Wednesday, by Primary Dealers. We use the aggregated market value of positions for four maturity groups of securities: (1) remaining maturity less than or equal to 3 years, (2) remaining maturity greater than 3 years but less than 6 years, (3) remaining maturity greater than 6 years but less than or equal to 11 years, (4) remaining maturity greater than 11 years. For convenience, we will refer to these maturity groups as 2-year, 5-year, 10-year, and 20-year, which is similar to the maturities for the futures contracts, described next. Data are reported for these maturity groups until April 2013 but are reported in more disaggregated groups for later data. We retain the original grouping in order to obtain a longer sample. The data are for Treasury notes and bonds; they do not include Treasury bills, TIPS, agency securities, or other holdings.

The position data for futures is constructed from the daily Large Trader files reported by futures commission merchants to the U.S. Commodity Futures Trading Commission (CFTC). [1] . The relevant contracts are for nearby expiries on four underlying instruments: the 2-Year, 5-Year, and 10-Year Treasury notes, and for the Treasury bond. We use the nearby contract with the largest daily futures trading volume. Treasury futures market is a limit order market, where there are no designated intermediaries. However, participants in this market is classified as "Financial Dealers and Intermediaries (FDI) depending on their predominant business purpose. While this trader classification in the data is self-reported and iit s subject to review by CFTC staff for reasonableness. [2] The list of market participants in the FDI category is closely related to the list of Treasury cash market dealers. Eventhough FDI traders do not have formal market-maker obligation in the futures market, we refer to them as "dealers" to be consistent with their activities in the Treasury cash market. We use positions held by accounts designated as FDI to measure the dealer positions in the futures market. The data includes futures positions and delta-adjusted options positions; we generally refer to these data as futures for brevity. Dealers report their Treasury cash positions in terms of market value while reported dealer positions in the futures market are in number of contracts. We convert the dealer positions in futures to equivalent market values by multiplying the number of contracts with the prices of the cheapest-to-deliver Treasury security prices. We sample this data weekly, on Wednesdays.

[1] CFTC releases the public version of this data, which is called Commitment of Traders (COT). The COT data is reported weekly as of Tuesdays. For robustness analysis, we reran our analysis replacing the confidential CFTC data with the COT data assuming that COT data is valid for Wednesdays to match with the Cash market data. The results are qualitatively the same as reported in the paper.

[2] https://www.cftc.gov/MarketReports/CommitmentsofTraders/index.htm

Table I reports summary statistics of key variables. Several observations are evident from the table. First, mean cash Treasury inventories are negative for the shorter maturity buckets, but the mean is positive for the longest maturity bucket ("20-year"). In contrast, mean dealer positions for futures are positive for the 5-year and 10-year buckets and negative for the longest maturity bucket. Net market values are, on average, negative in all maturity buckets. Second, the order flow variables usually have means with different signs across the cash and futures groups. Recall that this variable represents -1 times the weekly change in Dealer positions and therefore, positive (negative) values represent customer net buying (selling). Therefore, it appears that, on a week to week basis, flows in futures and cash offset somewhat for Dealers. Third, while there appears to be substantial persistence in the levels of Dealer inventories, there is a significant negative autocorrelation in weekly order flow. This negative autocorrelation holds for cash, futures, and net order flow.

Figure 1 displays the time series of net inventories of dealers, in terms of market value, in both the Treasury futures and cash markets. Each panel displays the net positions for a broad maturity bucket of positions (e.g., Treasury notes less than 3 years to maturity) and the related futures contract (e.g., the 2 Year futures contract). The magnitudes of the dealer positions in the cash and futures markets are generally comparable. Broadly, cash inventories tend to be on the short side for much of the sample up to around 2009 and have been toward the long side since then; futures positions show roughly the opposite pattern. Moreover, some of the shorter-term shifts in dealer cash inventories are mirrored by an opposite movement in futures positions. That is, local peaks (troughs) in cash market positions for a given bucket are roughly mirrored by troughs (peaks) in futures positions within that maturity bucket.

Visual inspection of the time series of cash and futures positions, across maturity buckets, also suggests that the offsetting nature of cash and futures positions is more evident at longer maturities. In particular, there appears to be a very weak relation between cash and futures positions for the 2-year maturity bucket. Further, while some general characteristics of positions appear across multiple maturity buckets, the groups do not appear to be perfectly correlated with each other. For example, while futures market positions in the 10-year bucket generally trended strongly upward during the 2004-2007 period, the bond futures position remained relatively flat.

We expect that dealers to stat flat relative to the factors, and that they hedge with derivatives on that basis. Therefore, we aim to re-express the Dealer position data in terms of risk factor exposures. To construct these factor values, we represent factor exposures in dollar terms rather than as a percentage of bond price, as Diebold, Ji and Li (2006) do. The resulting risk factor values represent the dollar risk to the portfolio holder, due to a one unit shock in a given risk factor. The details of this construction are explained in the appendix A. Figure 2 displays the net dealer positions in cash and futures (across all maturities), decomposed into three portfolios representing the dollar exposure to the level, slope, and curvature risk factors, similar to those derived by Diebold, Ji and Li (2006).

We make two main conclusions from examining Figure 2, in which we display the computed risk factors. First, we find that the majority of the risk exposure appears to be accounted for by a single factor. That is, the risk in a given market is quite similar whether it is measured by level, slope, or curvature; the risk from a given factor is highly correlated with the dollar risk from the other two factors (correlations above 0.95). Second, the pattern of risk exposures over time is not surprising, given the market values from the initial presentation of the data. Dealers, in aggregate, were net short Treasury risk in the cash market during the 2001-2008 period and generally long Treasury cash market risk during the 2009-2018 period; they held futures market positions that generally took the opposite sign during these broad periods. This observation was true for the level factor, as well as the other factors.

The last step in summarizing the raw data is to compare the cash and futures positions in a regression framework. Table II presents results from this exercise. Panel A displays the regression of each futures market factor exposure on the corresponding, contemporaneous cash market factor exposure. The results are quite consistent with Dealers using futures markets to offset their cash market risk. Futures risk exposure is decreasing in cash market exposure for each of the three factors, and the results are significant at any conventional significance level. The regression R^2 measures are all above 70%. However, the slope coefficients are far away from unity, suggesting that the offset is not one-to-one. Further, the significant intercepts suggest that Dealers do not simply mechanically offset cash market exposure with futures market positions. This is consistent with the results of Fleming and Rosenberg (2008), who find that Dealers appear to hedge some positions but do not appear to hedge new issues taken on via the Treasury issuance process.

Panel B presents results from a first differenced version of the factor regressions. These results suggest that week-to-week changes in futures position risk exposures are significantly negatively correlated with changes in cash market position risk exposures. The slope coefficients are far from unity, again suggesting that the offset is not a mechanical, one-to-one offset. The R^2 value is 7.0% for the level factor and 4.5% for both the slope and curvature regressions.

The regression results are robust, as we have obtained very similar results with other specifications (not shown here). We have estimated regressions of futures exposure on cash exposure using market values of each and one regression for each of the four maturity buckets shown in Figure 1. In that case, we obtained results broadly similar to the ones shown in Table II, but the slope coefficients for the 2-year maturity bucket are much weaker than for the other buckets (e.g., the R^2 was 6% in the 2-year bucket levels regression but 4-8 times that for the other buckets). Another variation featured the same regression model, but inventories were measured in DV01 rather than market values. The results were quite similar for this specification: the slope coefficients were reliably negative for both the levels and first difference regressions. As before, the fit for the 2-year bucket was not nearly as good as for the longer maturity buckets (e.g., the R^2 values are 7-10 times higher for the longer maturity buckets for the level regressions and the result is even stronger for the first difference regressions).

V. Estimation of the State Space Model

We estimate the State-space model specified in equations (1) through (3). As in Diebold, Rudebusch and Aruoba (2006), we estimate this state-space model with a Kalman filter. Our estimation strategy is to begin with simplified versions of the model and progressively estimate more complex specifications.

We first convert the market value of dealer inventories to a dollar-value-of-a-basis-point (DV01) measure.[3] For cash positions, we compute DV01 by assuming that the market value is held in a representative bond for each maturity bucket. We assume that the representative bond has a maturity of the midpoint of the bucket (e.g., a 4.5 year maturity bond for the 3 to 6 year bond group), a coupon equal to the weighted average coupon associated with the Citigroup Benchmark Government Bond Index, and we interpolate the market yield from Federal Reserve H15 constant maturity yields. We then compute the DV01 analytically using a linear approximation. For futures positions, we rely on the characteristics of the cheapest-to-deliver bond on each date, as given by

[3] Although it would be theoretically consistent to model inventory risk using the portfolio exposures to the level, slope, and curvature factors, as estimated in the previous section, we faced collinearity issues in estimating and interpreting the results, due to the large number of parameters. This issue was exacerbated as we moved to larger models. Given the overwhelming importance of the level factor in the previous results, we focus on DV01 as a practical measure with transparent interpretation.

Bloomberg. We use the exact maturity, coupon, and full-price yield of this bond to compute the cash DV01 by revaluing the bond at varying yields and then divide this value by the associated futures conversion factor provided by the exchange.

We choose to measure inventory $I_t(\tau)$ in terms of DV01 for its ease of interpretation, across maturity buckets and time, as a risk measure; however, we recognize that other choices are plausible. The raw data is in market value of positions, but using market values would obscure the risk of inventory across different maturities. Another alternative is to convert market values into estimated face values, but that measure ignores the variation in inventory risk across time and in the cross-section. In computing the DV01 for cash bonds, our estimate utilizes H15 yields rather than Gürkaynak, Sack and Wright (2007), because these values incorporate both off-the-run and on-the-run bonds. We believe these yields are likely to be more representative than the off-the-run yields from Gürkaynak, Sack and Wright (2007). Nonetheless, we have estimated the model with all of these variations (including market value and face value of inventories), and the results are qualitatively similar to the ones presented here.

After converting to DV01, we find that there is substantially more risk in the longer-dated buckets than in the 2-year bucket. On average, the absolute value of the 2-year bucket DV01 is USD 2-3 million for both cash and futures, whereas the absolute value of DV01 for the longer-dated buckets are in the USD 6-14 million range. For both cash and futures markets, the average of the absolute values of the DV01s are monotonically increasing in maturity of the buckets. Specifically, we find that the absolute value of the cash DV01 ranges from USD 3.5 million in the 2-year bucket to USD 13.8 million in the long bond bucket, while the futures DV01 ranges from USD 1.8 million in the 2-year bucket to 10.3 million in the long bond bucket.

With respect to units, we measure Treasury yields in basis points and the DV01 of dealers in millions of US dollars. Because the raw non-dealer order flow variables feature such negative autocorrelation, as shown in Table I, we pre-filter the order flow by using the residuals from a regression of raw order flow on one lag of itself. We therefore interpret the order flows as surprises or innovations to order flow, but we simply refer to them as "order flow" for brevity. Order flow values are measured in billions of US dollars. In the estimation, we maximize the log-likelihood function with the Kalman filter initialized with diffuse prior distributions for parameters in the state and observation equations. These diffuse quantities are treated as zero-mean and Gaussian random variables.[4]

A. Price Pressure

Our baseline model allows for price pressure effect for the net DV01 of Dealers, but no price discovery effect. Estimation results for our baseline model are reported in Table III. Panel A reports the four coefficients that reflect the impact, in basis points, of dealers holding one million USD of DV01 within each maturity bucket. We refer to this as the "Net Inventory" model, because the DV01 values used in estimation represent the net DV01 (cash exposure plus futures exposure) for dealers, within a given maturity bucket. The model includes the three latent factors, as described in Equation (4). Each of these three factors are entirely latent in this estimation; they are AR(1) processes with no explanatory variables included. In each case, the variables display a very slight amount of mean reversion: the estimated autoregression coefficients are above 0.99. Innovations to the factors are virtually permanent.

Of the four "price pressure" coefficients in Panel A, three are quite significant at conventional levels. The coefficients are positive, which is consistent with dealer long inventory depressing prices of the maturity segment associated with the inventory bonds, and therefore raising yields. The coefficient on the 2-year net inventory is insignificant

[4]The state-space model is estimated with active-set optimization method in SAS.

and virtually zero, but the coefficients on the other maturity buckets appear quite important. The magnitudes suggest that each USD million of long DV01 in each maturity bucket increases the Treasury yield by roughly a quarter to a half a basis point. This is strong evidence for a "price pressure" effect of dealer inventory on Treasury yields.

Given the simple structure of the model, we can readily compute the net effect of dealer inventory for each maturity bucket, at each date t, by multiplying the relevant parameter by the net inventory. These values are plotted in Figure 3. Given the very small value of the 2-year inventory parameter, the plot for that maturity is virtually invisible, suggesting an economically insignificant effect. However, for the other maturity buckets, the effect is readily visible. Furthermore, their average effect, measured as the product of the parameter estimate and the standard deviation of the comparable maturity Dealer DV01, is economically significant. For example, the price pressure for the 5-year maturity bucket averages 1.85 basis points, and the average effects for the 10-year and 20-year buckets are roughly 5 basis points in both instances. The averages do mask significant variation, as the 10-year bucket price pressure effect displays the most volatility of the series, dipping to nearly -30 basis points in late 2012. The price pressure effect for the 20-year bucket was typically negative and in the low single digits for most of the sample, but it has trended upwards toward 10 basis points during the 2013-2018 portion of the sample.

This result contrasts with the findings in Brandt and Kavajecz (2004), who find no compelling evidence that the daily variation in Treasury yields is due to inventory effects, and therefore ascribe the variation to price discovery. Fleming and Rosenberg (2008) find that the relationship between dealer positions and Treasury prices is, on average, inconsistent with dealers receiving compensation for holding risky inventory, except for short periods around Treasury issuance. We reconcile our findings with prior results by noting that our model allows us to focus on the level of dealer inventory, which is highly persistent. The daily or weekly variation in the pricing of inventory risk is likely to be minuscule in most instances, but we are able to identify this economically material effect of the level of inventories.

A.1. Cash vs Futures Price Pressure

We can readily extend the model to address the question whether the impact of dealer inventory varies if the exposure is held via cash or futures. While we expect the instruments to be close substitutes, there are a few reasons to believe that they do not exactly offset each other in dealer inventory.

First, futures are standardized in just a few instruments whereas particular Treasury cash securities may not be as liquid due to their specialness (e.g., an off-the run note versus an on-the-run note). Second, futures market is centralized while cash Treasury trading is fragmented across venues and over-the-counter markets, imposing search costs for market participants. Third, regulations may not affect cash and futures markets uniformly. For example, Duffie (2017) argues that supplementary leverage ratio (SLR) impacts repo rates. Repo rates could affect the financing rates of cash dealer inventories. On the other hand, SLR also affects the Treasury futures market, because Treasury futures positions are included in the calculation of SLR for banks. Hence, it is not clear whether futures exposure or cash exposure is costlier in dealer inventory.

In order to test this which instrument is costlier in dealer inventory, we extend the baseline model by allowing market yields to depend (for each maturity bucket) on the aggregate DV01 held in cash instruments and the aggregate DV01 in futures.

Table IV displays the results of our extended model. Our first observation is that all of the coefficients for the 5-year, 10-year, and 20-year buckets are positive and statistically significant, which is consistent with the baseline results and the intuition that dealer inventory raises yields. When we compare the futures and cash coefficients for a given maturity, we find evidence that futures exposure has less of an impact on inventory than the equivalent DV01 in cash instruments. In particular, the coefficient on the 10-year cash DV01 is 0.59, while the coefficient on the 10-year futures is 0.51, and the difference between them is statistically significant. Although the prior results indicate that the 2-year bucket is not nearly as important relative to the longer-dated buckets, the coefficients for the 2-year bucket are of opposing signs for cash and futures DV01 with difference statistically significant. We do not reject equality of the cash and futures coefficients in 5- and 20-year maturities. Taken together, we conclude that futures and cash exposures are not perfect substitutes, and that cash bonds held in inventory exert more of an impact than the equivalent DV01 of futures exposure. This effect manifests itself at the 2-year and 10-year maturities.

B. Price Discovery

In this section, we add the order flow variables into the state-space model to explore their price discovery implications. While the baseline model and its extension featured latent level, slope, and curvature factors to describe the common dynamics of interest rates, we now allow innovations to the level and slope curvature factor to include innovations to order flow. In this way, we allow a correlation between customer buying interest in Treasury exposure and Treasury yield changes. Our modeling approach ties order flow to factors, rather than specific maturities or instruments. This generality of having price discovery take place at the factor level allows the model to distribute the impact of order flow to related instruments without resorting to ad hoc methods.

Panel B of Table V reports the price discovery coefficients when the level and slope factor levels include the order flow variables for the 2-year, 5-year, 10-year, and 20-year maturity bucket order flows, as in equation (4). Given the relatively small set of yields used in estimation, we experienced collinearity problems when attempting to include order flow in the curvature factor. Therefore, we maintain the curvature factor as a purely latent factor, with no observable variables included.

Of the eight price discovery coefficients now included, we find three of them to be quite statistically significant. In the level factor, we find that the 10-year order flow is highly significant, with a t-statistic of 3.57. The sign is negative, suggesting that customer net buying of 10-year note exposure is associated with a decline in the level factor, which is intuitive. However, because the 10-year order flow is in both the level and slope factor, the coefficient cannot be interpreted as a marginal effect without further analysis. We also find that the order flow coefficients for the 5-year bucket and the 10-year bucket are quite significant (t-statistics of -2.22 and 4.28, respectively) in the slope factor equation. Subject to the interpretation difficulty signaled above, the coefficients have opposing signs, which is superficially suggestive that order flow can be associated with a twist in the yield curve.

In order to interpret these price discovery coefficients better, we can compute comparative statics using the estimated parameters. The goal is to trace through the change in yield, across the entire term structure, given an innovation in net orderflow at a particular maturity. If we assume a one standard deviation shock to the 10-year orderflow (USD 9.4 billion), the associated effect is the depress yields across most of the term structure, with larger impacts at longer maturities. Specifically, the 20-year yield declines 1.7 basis points, the 10-year declines 1.4 basis points, and the 5-year declines 0.7 basis points. Mechanically, this is because the impact of the level effect coefficient is offset to a large extent by the slope effect coefficient at shorter maturities (where the slope factor loading is relatively high), but the level effect dominates at longer maturities (where the slope factor loading is relatively low).

Next, we perform the same exercise to understand the marginal effect of net orderflow in the 5-year bucket. If we assume a one standard deviation shock to 5-year net orderflow (USD 6.9 billion), such net customer buying is associated with a decline across the term structure. The largest effect is on the 2-year yield, which declines 1.8 basis points; the 5-year declines 1.2 basis points, and the 10- and 20-year decline 0.9 and 0.8 basis points, respectively. Mechanically, this reflects the negative coefficients for the 5-year order flow in both the level and slope factors. The innovation is associated with a downward move in both factors, although the decline in the slope factor loading at longer maturities means that the slope factor exerts less influence at those maturities.

Table VI displays another measure of the influence of order flow that is consistent with statistical significance but provides more context. Recall that equation (4) allows the latent factors to depend on the lagged value of the latent factor, the order flow, and idiosyncratic noise. Similar to the measure suggested by Hendershott and Menkveld (2014), we compare the variation in the product of the order flow and the impact coefficient with the variation of the idiosyncratic noise for that latent factor. This calculation yields four ratios per factor: one for each order flow. For the level factor, we find that the largest ratio is for the 10-year order flow, and its value is 2.96%. The other values are well below 0.25%. For the slope factor, we find that the largest ratio is for the 10-year order flow (at 6.71%) and the second largest is for the 5-year order flow (at 1.17%). The other values are far smaller.

This analysis suggests that a small fraction of the innovation in the factors is related to order flow. Nonetheless, we stress that these order flow values are weekly measures of customer net buying. Whereas order flow is typically measured in intraday or daily intervals, we believe the significance and reasonableness of the estimated relations are quite striking, given the very long timescale we are using compared to the literaure.

Finally, we estimate another extension of the basic model in order to isolate the source of the price discovery just established. There is a longstanding research interest in identifying, for related instruments, the market in which price discovery occurs. As noted previously, Brandt, Kavajecz and Underwood (2007) and Mizrach and Neely (2008) conclude that, while information is transmitted to prices from both markets, the cash markets are particularly important for price discovery at the short end of the curve, while futures markets are more important for price discovery at longer maturities. We extend our model to allow factor shocks to come from cash market order flow and/or from futures market order flow, rather than solely from order flow netted across the two markets. We focus on expanding the model for the three statistically significant coefficients identified in the previous estimation. Therefore, we allow the level factor to be impacted by cash and futures order flow in the 10-year maturity, although we retain net order flow for other maturities. For the slope factor, we allow the 5- and 10-year cash and futures order flows to have separate impacts. We retain net order flow for the 2- and 20-year buckets.

Table VII displays the results of this extended model. For the level factor, we find that the 10-year futures market order flow is statistically significant (t-statistic of -4.41), but the 10-year cash market order flow is not significant (t-statistic of -0.23). The other net order flow coefficients remain insignificant. For the slope factor, we find that the 5-year order flow for the cash market appears quite important (t-statistic of -2.67), but the 5-year futures order flow is not. We also find that the 10-year order flow in the cash market is unimportant for the slope factor, but the 10-year futures market orderflow is important (t-statistic of 3.67). We conclude that the model successfully isolated the markets in which primary price discovery occurs: the 5-year cash market and the 10-year futures market.

As described above, it is useful to perform comparative statics exercises on the models to evaluate the marginal effects of a shock, because the parameters themselves should not be interpreted as marginal effects. As before, we gauge the impact of a one standard deviation shock to a given orderflow and trace out the associated yield curve shifts. The top panel of Figure 4 displays these shifts in the yield curve in response to a one standard deviation shock to the 5-year cash and futures order flows. We find that a one standard deviation shock to the cash market order flow in the 5-year maturity is associated with a decline in yields across the term structure, with the largest effect at the front end of the curve. The implied change in yield is -1.7 basis points for the 2-year maturity, monotonically declining in magnitude to 0.5 basis points for the 20-year yield. In contrast, a one standard deviation shock to order flow in the 5-year Treasury futures generates a similar pattern, but with roughly half of the magnitude at the front of the curve. The 2 year yield declines by 0.9 basis points and the effect tapers off to 0.4 basis points for the 20 year yield. Net customer buying in the futures market has a much smaller impact on the curve than net customer buying in the cash market.

The bottom panel of Figure 4 displays the behavior of the yield curve in response to a one standard deviation shock to the 10-year cash and futures order flows. We find even more dramatic differences across the markets. A one standard deviation shock to 10-year futures order flow is correlated with a 1.8 basis point decline at the back end of the curve that tapers to 0.3 basis points at the front of the curve. However, there is no meaningful relation between 10 year cash market order flow and yields. The associated yield decline is less than 0.1 basis point across the term structure. Net customer buying in the 10-year futures market has a strong impact on the term structure, especially at the back end, but net customer buying in the 10-year cash market has virtually none.

VI. Conclusion

Our goal in this paper is to analyze the effects of dealer inventory and order flow on Treasury yields by using a term structure model. Our strategy is to modify the term structure model to allow for mispricing to depend on dealer inventory and the factors governing the yield curve to be impacted by order flow. The resulting specification is flexible enough to estimate the price pressure and price discovery effects of different securities across different maturities. We use this flexibility to investigate the relative importance of Treasury cash and futures markets across different maturities for price discovery and liquidity provision.

We build on market microstructure models of inventory and price discovery, such as Hendershott and Menkveld (2014), in order to separate fundamental and non-fundamental drivers of prices. We rely on the dynamic Nelson-Siegel term structure factor model specified in Diebold, Rudebusch and Aruoba (2006) and related papers in order to estimate fundamental moves and to distribute them in a given market or maturity point to related markets or maturities.

We have three main findings. First, we find a statistically significant effect of dealer inventory of specific maturities on yields of Treasury securities with a similar maturity. Net positive (negative) dealer inventories, where inventories are defined as the sum of cash and futures positions, are associated with higher (lower) market yields. This finding is consistent with dealers providing liquidity to the market in return for compensation through price concessions: a "price pressure" effect. The net effect on yields varies across the term structure and over time. For example, we estimate that dealers were typically short Treasury exposure during the 2001-2013 period, and we estimate that this behavior decreased market yields in the 10-year yield by nearly 5 basis points, on average. Second, we find evidence that long-dated interest rate exposure via cash instruments is associated with a larger inventory effect on yields than exposure to long-dated futures. We conclude that this supports the idea that cash and futures are not perfect substitutes in a dealer book. Third, our model accommodates a price discovery channel by linking non-dealer order flow to fundamental moves in the yield curve. Consistent with this channel, we find a significant link between order flow and latent factors that describe bond yield changes. Specifically, the links are strongest between order flow in the 5-year cash Treasury and movements in the front end of the curve and between order flow in the 10 year Treasury future and movements in the back end of the curve.

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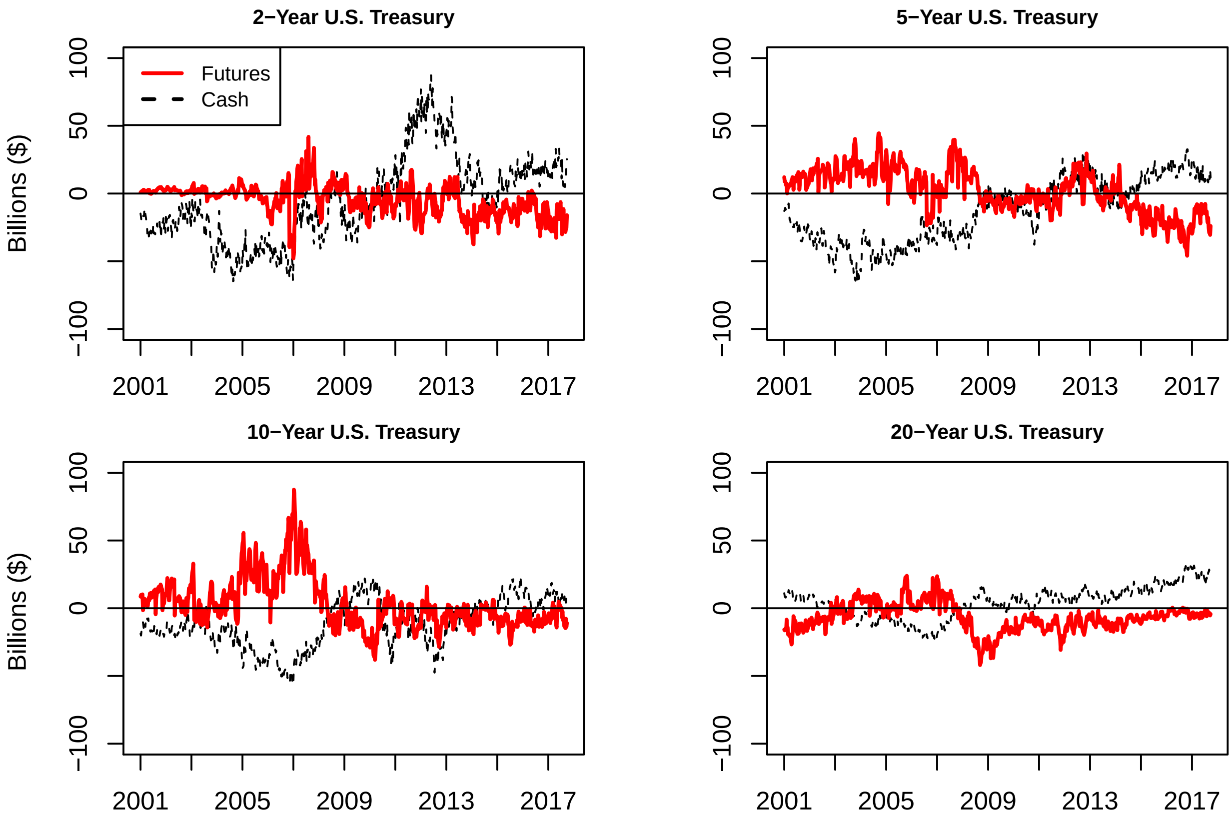
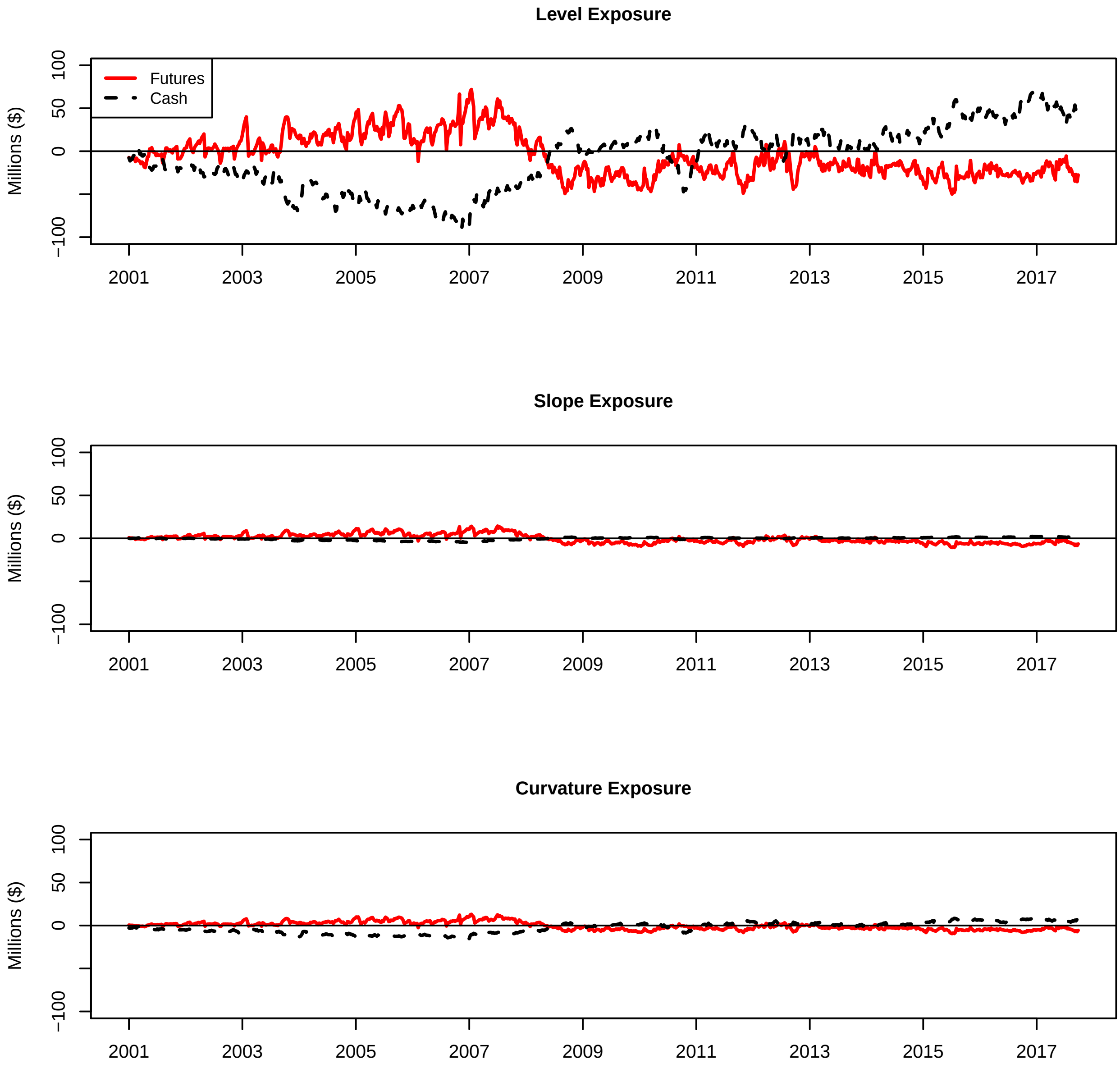
Figure 1: Dealer Treasury Inventories, by Maturity



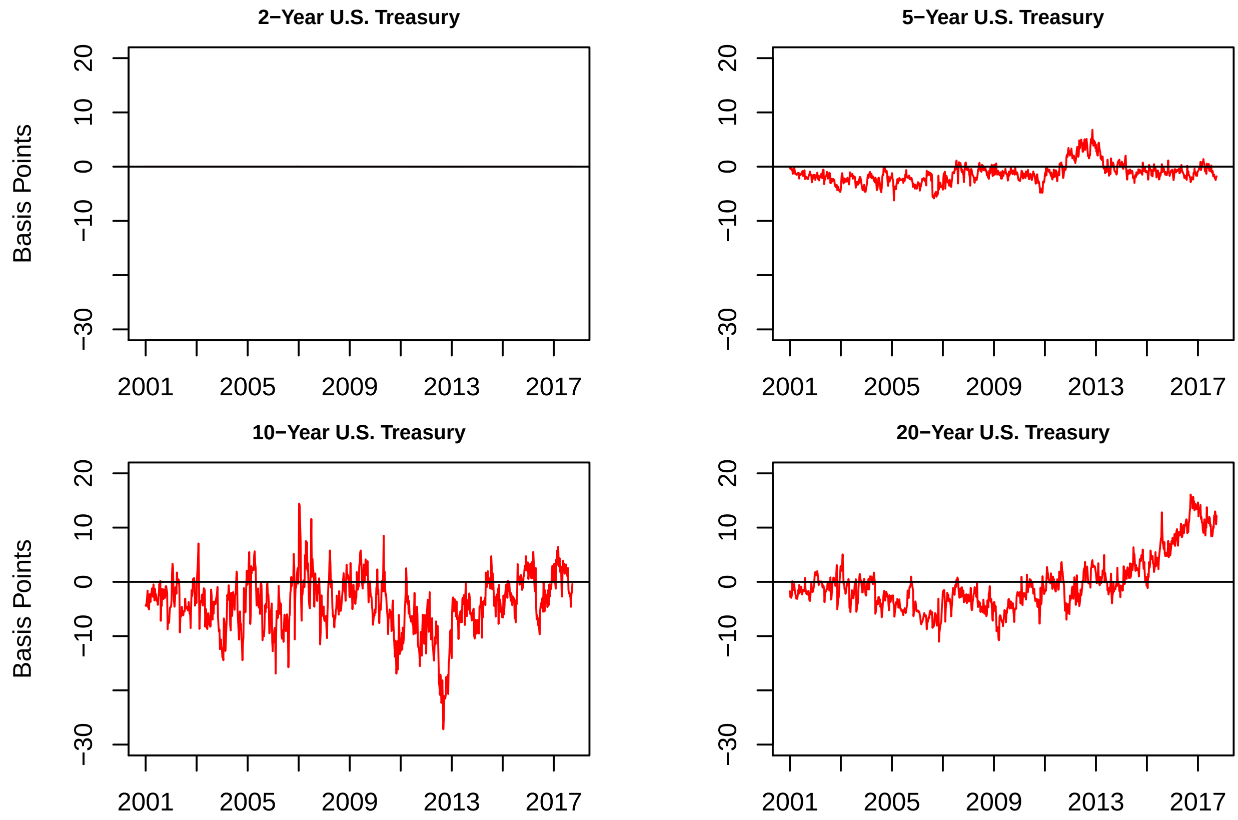
This figure plots the weekly dealer inventories in the Treasury futures and cash markets. Dealer Treasury futures positions are aggregated trader positions in the financial intermediary and dealers (FDI) category in the front-month futures contracts. Dealer Treasury cash positions are the aggregate market value positions of primary Dealers published on the Federal Reserve Bank of New York's website. Futures positions in 2-, 5-, 10-, and 20-year U.S. Treasuries are defined as front-month positions in 2-, 5-, and 10-year Treasury note futures, and Treasury bond futures, respectively. Treasury futures positions are multiplied with the prices of the cheapest-to-deliver bond prices to represent market values. Cash positions in 2-, 5-, 10-, and 20-year U.S. Treasuries are defined as positions with remaining maturities of more than 2 years but less than 3 years, remaining maturities of more than 3 years but less than 6 years, maturities of more than 7 years but less than 11 years, and remaining maturities of more than 11 years, respectively.

Figure 2: Dealer Treasury Inventory Risk, by Factor



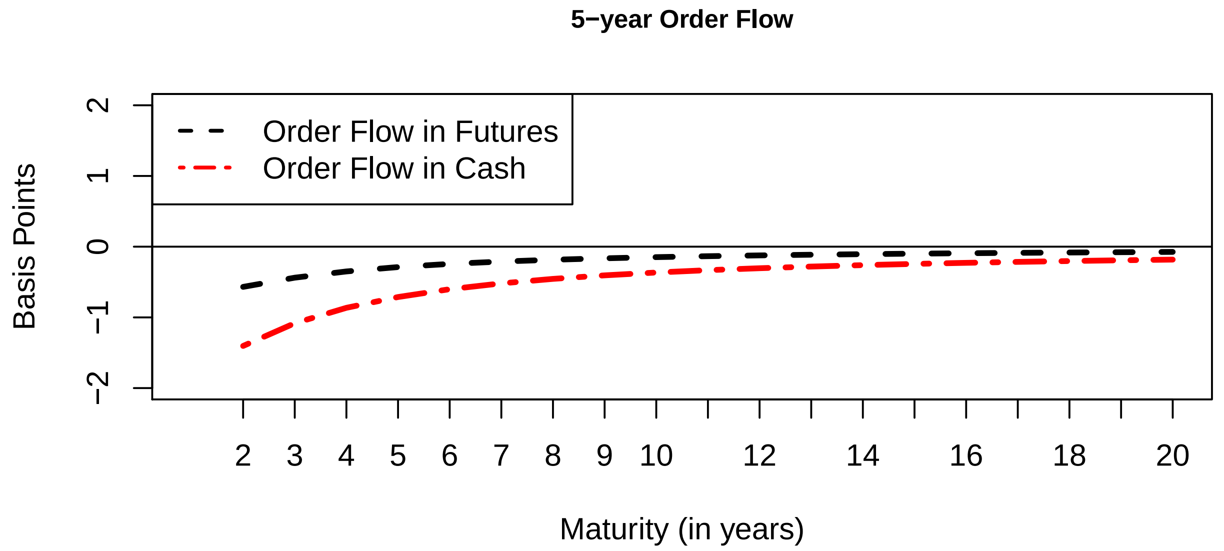
The figures display the dollar risk to Dealer inventories in response to a 1 basis point shock in the Level, Slope, and Curvature Factors. These factor exposures are computed as similar to Diebold, Ji and Li (2006).

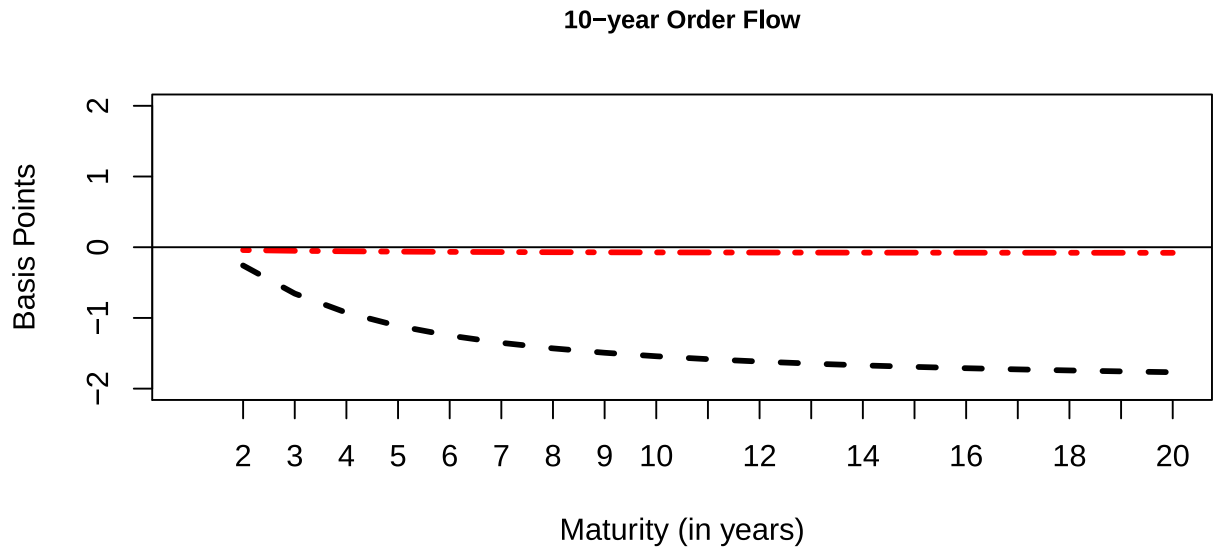
Figure 3: Estimated Price Pressure Effect on Treasury Yields



The figures display the product of Dealer net DV01 (cash positions and futures positions combined) and the estimated "price presure" coefficients from table III, which relates Dealer DV01 at a given maturity to Treasury yields. The value represents the marginal impact of aggregate Dealer DV01 in each maturity bucket on Treasury yields.

Figure 4: Impact of a 1-std shock to the Order Flow on the Term Structure Curve





The figures display the impact of 1 standard deviation shock to the order flow on the yield curve using the standard deviations from table I and the estimates from table VII.

Table I: Summary Statistics

	Yield (%)	Dealer Inventories ($ Billion)			Order Flow ($ Billion)		
		Cash	Futures	Net	Cash	Futures	Net
MEAN							
2-year	1.78	-5.13	-4.96	-10.09	-0.05	0.02	-0.03
5-year	2.58	-13.53	2.38	-11.15	-0.03	0.04	0.01
10-year	3.51	-11.63	3.08	-8.55	-0.04	0.02	-0.03
20-year	4.15	4.33	-6.91	-2.58	-0.02	-0.01	-0.03
MEDIAN							
2-year	1.27	-8.63	-2.55	-13.20	0.04	0.02	0.22
5-year	2.34	-10.69	2.74	-11.36	0.12	0.04	0.03
10-year	3.69	-10.98	-1.16	-7.91	0.08	-0.15	-0.02
20-year	4.47	6.11	-6.96	-4.17	0.11	-0.08	-0.12
STD							
2-year	1.46	31.48	11.43	30.73	7.27	5.70	8.96
5-year	1.25	23.36	16.17	16.52	5.04	5.41	6.88
10-year	1.15	17.53	18.54	13.16	4.24	6.23	9.42
20-year	1.10	11.92	10.26	11.05	2.09	3.33	3.43
$\rho(1)$							
2-year	0.996	0.974	0.876	0.958	-0.303	-0.178	-0.245
5-year	0.993	0.977	0.945	0.913	-0.330	-0.156	-0.251
10-year	0.992	0.972	0.944	0.860	-0.109	-0.072	-0.213
20-year	0.994	0.986	0.947	0.954	-0.115	-0.156	-0.157
N	872	872	872	872	872	872	872

The table reports the summary statistics for the 2-, 5-, 10 and 20-year zero coupon Treasury yields and the weekly dealer inventories in the Treasury market. Net inventories are the sum of dealer inventories in Treasury cash and futures markets. Futures positions in 2-, 5-, 10-, 20- US Treasuries are defined as front-month positions in 2-year, 5-year, 10-year Treasury note futures, and Treasury bond futures, respectively. Treasury futures positions are multiplied with the prices of the cheapest-to-deliver bond prices to represent market values. Dealer Treasury cash positions are the aggregate market value positions of Primary Dealers published on NY FED website. Cash positions in 2-, 5-, 10, 20- US Treasuries are defined as positions with remaining maturities more than 2 years but less than 3 years, remaining maturities more than 3 years but less than 6 years, maturities more than 7 years but less than 11 years, and maturities more than 11 years, respectively. Order Flow is defined as the negative of the weekly change in Dealer inventories. $\rho(1)$ is the coefficient on the lagged term from an AR(1) regression.

Table II: Regressions of Futures Inventory on Cash Inventory, by Factor Exposure

Panel A

	$Level^{Fut}$	$Slope^{Fut}$	$Curve^{Fut}$
α	-10.03	-1.43	-2.00
	(0.44)	(0.09)	(0.08)
$Level^{Cash}$	-0.55		
	(0.01)		
$Slope^{Cash}$		-2.74	
		(0.06)	
$Curve^{Cash}$			-0.63
			(0.01)
# of obs.	872	872	872
$Adj-R^2$	72.5%	72.0%	74.7%

Panel B

	$\Delta Level^{Fut}$	$\Delta Slope^{Fut}$	$\Delta Curve^{Fut}$
α	0.01	0.00	0.00
	(0.23)	(0.05)	(0.04)
$\Delta Level^{Cash}$	-0.42		
	(0.05)		
$\Delta Slope^{Cash}$		-1.74	
		(0.27)	
$\Delta Curve^{Cash}$			-0.35
			(0.05)
# of obs.	871	871	871
$Adj-R^2$	7.0%	4.5%	4.5%

This table reports the regression of Dealer futures inventory factor exposure on Dealer cash inventory factor exposure. The dollar risk to Dealer inventories in response to a 1 basis point shock in the Level, Slope, and Curvature Factors are computed as similar to Diebold, Ji and Li (2006). Standard errors are in parentheses.

Table III: Estimation Results: Net Inventory Model

Panel A: Price Pressure Effect

	Estimate	t-value
π_{2-year}	0.00	-0.01
π_{5-year}	0.26	6.12
$\pi_{10-year}$	0.58	17.85
$\pi_{20-year}$	0.37	9.96

Panel B: Price Discovery Effect

	Level	Slope	Curvature
Θ	0.999	0.994	0.998
	(210.78)	(235.07)	(266.04)
Λ_{2-year}			
Λ_{5-year}			
$\Lambda_{10-year}$			
$\Lambda_{20-year}$			
Model AIC	25,528.44		

This table reports the estimation results of the model:

$$\widehat{y_t}(\tau) = y_t(\tau) + \pi_{\tau,\tau} DV01_t(\tau) + \epsilon_t(\tau)$$

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau} - e^{-\delta\tau}\right)$$

$$(\beta_t - \mu) = \Theta(\beta_{t-1} - \mu) + \omega_t$$

The model is estimated with Kalman filter, which is initialized with diffuse priors. δ is set to 0.0609. τ= 24, 60, 120 and 240 months. t-statistics of the estimates from the state equation are reported in parentheses below the coefficients.

Table IV: Estimation Results: Cash and Futures Inventory Model

Panel A: Price Pressure Effect		
	Estimate	t-value
π^{Cash}_{2-year}	0.84	4.39
$\pi^{Futures}_{2-year}$	-0.73	-4.07
π^{Cash}_{5-year}	0.23	3.92
$\pi^{Futures}_{5-year}$	0.30	5.62
$\pi^{Cash}_{10-year}$	0.59	16.39
$\pi^{Futures}_{10-year}$	0.51	11.04
$\pi^{Cash}_{20-year}$	0.43	10.12
$\pi^{Futures}_{20-year}$	0.39	7.61

Panel B: Price Discovery Effect			
	Level	Slope	Curvature
Θ	0.999 (203.58)	0.995 (241.09)	0.998 (281.85)
Λ_{2-year}			
Λ_{5-year}			
$\Lambda_{10-year}$			
$\Lambda_{20-year}$			
Model AIC		25,495.19	

This table reports the estimation results of the model:

$$\widehat{y}_t(\tau) = y_t(\tau) + \pi^{Cash}_{\tau,\tau} DV01^{Cash}_t(\tau) + \pi^{Futures}_{\tau,\tau} DV01^{Futures}_t(\tau) + \epsilon_t(\tau)$$

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau} - e^{-\delta\tau}\right)$$

$$(\beta_t - \mu) = \Theta(\beta_{t-1} - \mu) + \omega_t$$

The model is estimated with Kalman filter, which is initialized with diffuse priors. δ is set to 0.0609. τ= 24, 60, 120 and 240 months. t-statistics of the estimates from the state equation are reported in parentheses below the coefficients.

Table V: Estimation Results: Net Orderflow Model

Panel A: Price Pressure Effect			
		Estimate	t-value
π^{Cash}_{2-year}		1.36	4.66
$\pi^{Futures}_{2-year}$		-0.79	-3.06
π^{Cash}_{5-year}		0.28	4.47
$\pi^{Futures}_{5-year}$		0.32	5.53
$\pi^{Cash}_{10-year}$		0.58	16.13
$\pi^{Futures}_{10-year}$		0.50	10.66
$\pi^{Cash}_{20-year}$		0.42	9.50
$\pi^{Futures}_{20-year}$		0.36	6.40
Panel B: Price Discovery Effect			
	Level	Slope	Curvature
Θ	0.9999	0.9950	0.9980
	(79.87)	(241.53)	(285.99)
Λ_{2-year}	0.06	-0.17	
	(0.95)	(-0.88)	
Λ_{5-year}	-0.09	**-0.32**	
	(-1.31)	**(-2.22)**	
$\Lambda_{10-year}$	**-0.22**	**0.56**	
	(-3.57)	**(4.28)**	
$\Lambda_{20-year}$	-0.12	0.14	
	(-0.86)	(0.56)	
Model AIC		25,471.58	

This table reports the estimation results of the model:

$$\widehat{y_t}(\tau) = y_t(\tau) + \pi^{Cash}_{\tau,\tau} DV01^{Cash}_t(\tau) + \pi^{Futures}_{\tau,\tau} DV01^{Futures}_t(\tau) + \epsilon_t(\tau)$$

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau} - e^{-\delta\tau}\right)$$

$$(\beta_t - \mu) = \Theta(\beta_{t-1} - \mu) + \Lambda OF_t + \omega_t$$

The model allows for OF_t to have an impact on Level and Slope factors only. The model is estimated with Kalman filter, which is initialized with diffuse priors. δ is set to 0.0609. τ= 24, 60, 120 and 240 months. t-statistics of the estimates from the state equation are reported in parentheses below the coefficients .

Table VI: Price Discovery: Orderflow Importance in Factor Variation

Ratio of Variation due to Orderflow vs. Latent Innovation ($\frac{\boldsymbol{\Lambda^2 \times Var(OF)}}{\boldsymbol{Var(\omega)}}$)		
	Level	Slope
$OF(\tau = 2yr)$	0.21%	0.54%
$OF(\tau = 5yr)$	0.23%	1.17%
$OF(\tau = 10yr)$	2.96%	6.71%
$OF(\tau = 20yr)$	0.11%	0.06%

This table reports the ratio of factor variation due to order flow as a ratio of latent factor innovation variance.

Table VII: Estimation Results: Cash and Futures Orderflow Model

Panel A: Price Pressure Effect

	Estimate	t-value
π^{Cash}_{2-year}	1.05	3.64
$\pi^{Futures}_{2-year}$	-0.42	-1.68
π^{Cash}_{5-year}	0.31	4.78
$\pi^{Futures}_{5-year}$	0.31	5.05
$\pi^{Cash}_{10-year}$	0.59	16.10
$\pi^{Futures}_{10-year}$	0.49	10.47
$\pi^{Cash}_{20-year}$	0.41	9.17
$\pi^{Futures}_{20-year}$	0.34	6.13

Panel B: Price Discovery Effect

	Level	Slope	Curvature
Θ	0.9999	0.9950	0.9980
	76.28	240.13	276.28
Λ_{2-year}	-0.07	0.18	
	(-1.34)	(1.05)	
Λ_{5-year}	-0.06		
	(-0.94)		
Λ^{Cash}_{5-year}		**-0.53**	
		(-2.67)	
$\Lambda^{Future}_{5-year}$		-0.20	
		(-1.09)	
$\Lambda^{Cash}_{10-year}$	-0.02	0.02	
	(-0.23)	(0.11)	
$\Lambda^{Future}_{10-year}$	**-0.32**	**0.53**	
	(-4.41)	**(3.67)**	
$\Lambda_{20-year}$	-0.07	0.13	
	(-0.46)	(0.52)	
Model AIC	25,469.53		

This table reports the estimation results of the model: $\widehat{y_t}(\tau) = y_t(\tau) + \pi^{Cash}_{\tau,\tau} DV01^{Cash}_t(\tau) + \pi^{Futures}_{\tau,\tau} DV01^{Futures}_t(\tau) + \epsilon_t(\tau)$; $y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\delta\tau}}{\delta\tau} - e^{-\delta\tau}\right)$; $(\beta_t - \mu) = \Theta(\beta_{t-1} - \mu) + \Lambda OF_t + \omega_t$ The model allows for OF_t to have an impact on Level and Slope factors only. OF variables with significant coefficients in table III are separated into cash and futures components. The model is estimated with Kalman filter, which is initialized with diffuse priors. δ is set to 0.0609. τ= 24, 60, 120 and 240 months. t-statistics of the estimates from the state equation are reported in parentheses below the coefficients.

Appendix A Risk Factor Exposure of Dealers

A bond price is the sum of discounted cash flows and is therefore a function of zero coupon yields. If zero coupon yields are a function of factors, then the dollar change in price of a given portfolio can be locally approximated as the sum of dollar price changes associated with a shock to each of the factors.

In equations, we begin by defining the time t price of a Treasury bond, P_t, as $P_t(\tau) = \sum_{i=1}^{I} c_i e^{-\tau_i y_t(\tau_i)}$. It follows that

$$dP_t(\tau) = \sum_{i=1}^{I} \left[\frac{\partial P_t(\tau)}{\partial y_t(\tau_i)}\right] dy_t(\tau_i) \tag{7}$$

From equation (3), it follows that $dy_t(\tau) = \Gamma_1 d\beta_{1t} + \Gamma_2 d\beta_{2t} + \Gamma_3 d\beta_{3t}$, where $\Gamma_1 = 1$, $\Gamma_2 = \frac{1-e^{-\delta\tau}}{\delta\tau}$, and $\Gamma_3 = \frac{1-e^{-\delta\tau}}{\delta\tau} - e^{-\delta\tau}$. Combining terms, we can express the absolute price change of a bond into its component risks:

$$|dP_t(\tau)| = \sum_{i=1}^{I} \left[c_i e^{-\tau_i y_t(\tau_i)} \tau_i\right] \sum_{j=1}^{3} \Gamma_j d\beta_{jt}. \tag{8}$$

Equation (8) can be written such that the relation of the risk factors to bond price changes is quite clear. Let $w_{it} = c_i e^{-\tau_i y_t(\tau_i)} \tau_i$, then

$$|dP_t(\tau)| = \sum_{i=1}^{I} w_{it} d\beta_{1t} + \sum_{i=1}^{I} w_{it}\Gamma_2 d\beta_{2t} + \sum_{i=1}^{I} w_{it}\Gamma_3 d\beta_{3t}. \tag{9}$$

In our implementation of these risk factors, we evaluate the risk exposure to the jth factor by setting $\beta_j = 1$ basis point and $\beta_{i\neq j} = 0$. Obviously, this is quite similar to the standard market practice of evaluating the dollar value of a basis point (DV01) for a bond portfolio; the similarity is especially close in the single risk factor case. We operationalize equation (8) by computing the risk factors for each maturity bucket and then summing the holdings of a given risk factor across maturities. For example, the dealer holding of slope risk is the computed slope risk for the 2-year bucket plus the slope risk for the 5-year bucket, 10-year bucket, and 20-year bucket, and so forth.

We assume that dealer cash inventories are held in a representative bond with a maturity equal to the midpoint of the relevant maturity bucket (e.g., the representative bond for the 3- to 6-year bucket has a maturity of 4.5 years), a coupon equal to the coupon value of the on-the-run index, and that zero coupon yields are given by the Gürkaynak, Sack and Wright (2007) model parameters for the relevant cash-flow date. Given the zero curve, coupon, and maturity, we can also estimate the face value of the bonds associated with the reported market value. To scale the value up to the portfolio level, we multiply the estimated risk value per dollar face value by the estimated face value of the bonds in the relevant maturity bucket. For futures, we compute the risk factors based on the cheapest-to-deliver bond for the relevant maturity bucket and then sum the holdings of a given risk factor across maturity buckets.